# REPORT RESUMES

ED 003 615 56

# PHYSICAL REQUIREMENTS FOR TEACHING MACHINES USING GRAPHIC DISPLAYS. FINAL REPORT, SUPPLEMENT.

BY- WAITE, D.P.
NORTHEASTERN UNIVERSITY, BOSTON, MASS.
REPORT NUMBER NDEA-VIIA-899-SUPPL PUB DATE JAN 63
GRANT OEG-7-31-0570-161
EDRS PRICE MF-$0.27 HC-$6.92 173P.

DESCRIPTORS- *GRAPHIC ARTS, *TEACHING MACHINES, *INSTRUCTIONAL MATERIALS, EVALUATION TECHNIQUES, *EQUIPMENT STANDARDS, TEACHING TECHNIQUES, *AUTOINSTRUCTIONAL AIDS, DESIGN, SECONDARY SCHOOLS, FILMS, *PROJECTION EQUIPMENT, *AUDIOVISUAL AIDS, BOSTON, MASSACHUSETTS, INFORMATION DYNAMICS CORPORATION

THE OPTICAL AND MECHANICAL REQUIREMENTS FOR GRAPHIC TEACHING MACHINES WERE DISCUSSED IN THIS SUPPLEMENT TO THE FINAL REPORT (ED 003 614). THE DESCRIPTIONS INCLUDED THE DESIGN OF AN AUTOINSTRUCTION SYSTEM, THE FUNDAMENTALS OF PROJECTION DISPLAY, MICROIMAGE PROGRAM MEDIA CHARACTERISTICS, USER NEEDS IN THE SECONDARY SCHOOLS, DETAILS OF EQUIPMENT DESIGN FACTORS, AND DETAILED CONSIDERATIONS IN PREPARATION OF PROGRAM FILMS. IT WAS RECOMMENDED THAT IN ADDITION TO MAKING AN EVALUATION OF THE SELF-INSTRUCTIONAL MATERIALS, AN EVALUATION OF THE MACHINE SHOULD ALSO BE MADE. (RS)

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
Office of Education
This document has been reproduced exactly as received from the person or organization originating it. Points of view or opinions stated do not necessarily represent official Office of Education position or policy.

SUPPLEMENT TO FINAL REPORT
for
Grant No. 7-31-0570-161

Dept. of Health, Education, and Welfare

# PHYSICAL REQUIREMENTS FOR TEACHING MACHINES USING GRAPHIC DISPLAYS, SUPPLEMENT TO FINAL REPORT

Report

to

Northeastern University
Boston, Massachusetts

from

Information Dynamics Corporation
2 Lakeside Office Park
Wakefield, Massachusetts

by

D. P. Waite
Principal Investigator

January 1963

ED003615

## TABLE OF CONTENTS

PREFACE

LIST OF ILLUSTRATIONS

CHAPTER Page

I. INTRODUCTION 1

II. AUTO INSTRUCTION SYSTEM DESIGN 5

- Introduction 5
- Designer's Scope 6
- Fundamentals of Graphic Information Presentation 9
- Considerations of Film Production 11
- Detailed Consideration of Equipment Design 11

III. THE FUNDAMENTALS OF PROJECTION DISPLAY 13

- Elements of an Optical Display System 13
- Basic Projection Optics 16
- Reflective Surfaces for Image Viewing 19
- Viewing Translucent Screen Images 21
- Display by Virtual Images 22
- Image Quality Factors 23

IV. MICRO IMAGE PROGRAM MEDIA CHARACTERISTICS 27

- Unitized vs. Roll Form 27
- Roll Form 27
- Flat Sheet Form 30
- Use of Standard Sizes 32

V. USER NEEDS IN THE SECONDARY SCHOOLS 34

- Graphic Content of Programs 34
- Further Factors in Graphic Presentations 37
- Environmental Requirements 40

VI. EQUIPMENT DESIGN FACTORS DETAILED 44

- The Design Analysis Diagram 44
- Environmental Factors 44
- Visual Effect Produced 46
- Image Projection System 50
- Film Copy Properties 55
- Insertion of Program Film 58

Page

Frame Locator Mechanism 59
Operator Controls 62
Student Response Provisions 63
Other Factors to be Considered 64

VII. DETAILED CONSIDERATIONS IN THE PREPARATION OF PROGRAM FILMS 66

Original Copy Preparation 66
Camera Operations 69
Film Processing 70
Error Edit and Splicing 71
Duplication and Distribution 71

VIII. SUMMARY 73

LIST OF REFERENCES

APPENDIX A - Use of the NBS Bar Chart for Resolution Measurements

APPENDIX B - ASA Standard for 35mm Sprocketed Roll Film

APPENDIX C - Results of Sampling Graphic Content of Secondary School Texts in Biology and Algebra

APPENDIX D - Sample of Graphics Illustrating Sample Categories

APPENDIX E - ASA Standard for Picture-Frame Sizes

APPENDIX F - Joint Statement on Self-Instructional Materials and Devices

GLOSSARY OF TERMS

## PREFACE

This report describes the results of a study to define the physical characteristics of teaching devices suitable for the display of graphic media in programmed learning. The work was done under contract with the Northeastern University in parallel with other studies in a broader project to develop requirements for graphic teaching machines.

The purpose of undertaking this study was to bring together for presentation in one report an analysis and discussion of the relevant factors of good engineering design. The intended use is primarily for users and workers in the field of auto instruction who are not widely versed in various engineering aspects of equipment design, yet who must have a basis for comparison and identification of those machines which meet their needs. It is the intention of the author to provide a lucid description of these design factors and an appraisal of alternatives open to the designer.

The secondary school system application of teaching machines is of primary interest. The benefits from the employment of automated teaching aids have already been demonstrated. There is little doubt about the potential pay-off for the newer field of teaching machines with their increased dimension of feedback in the student-machine relationship, which even more effectively imparts learning to the student. The number of machines that may be used in secondary school systems promises to be high, thus creating an attractive market for the manufacturers and producers of both teaching machines and programs for auto instruction. It is essential that some guide lines be established to channel the machine development efforts in order that the highest degree of compatibility between programs and machines be available. It makes little sense to consider a situation in which a school board is asked to authorize the purchase of a dozen different machine designs in order to gain access to the subject material offered by as many producers of programs.

It is not our objective here to design the optimum teaching machine within the present state of the technological art. There will never be complete agreement on what the optimum machine design is, and ingenuity and inventiveness of the designers of new machines should not be constrained by any one group's concept of the ultimate design. Further, one machine design will not meet all requirements. With technically sound guide lines established, however, presented in a manner which the non-technician can understand, this report should help clarify the user's understanding and his own definition of machine requirments. At the very least, it should assist in differentiating between machine design features which have been added to create uniqueness and prevent compatibility from those features which will truly achieve user benefit.

Another purpose of the report is to acquaint the technician of limited experience with the factors of machine design in the display area. Although ready-made components are available for assembly of simple configurations of photo-optical equipment individually suitable for teaching machine displays, the problem of combining them is not simple. It is hoped that the discussions given here will provide some insight and guidance to engineers experienced in other fields, such as electronics, who now find themselves facing the new assignment of tackling a complex machine design that integrates the problems of image formation, illumination levels, tight mechanical tolerances in film transports, electronic control, photography, properties of photographic film in combination with projection optics, and over-all awareness of user environment.

A clarifying phrase that should be used throughout this report is "visual display." The author recognizes that the visual is but one form of presentation of information in the learning process. A companion study would also be of value to present further design and equipment design factors for audio equipment.

Since it is assumed that the reader has more than a peripheral interest in the subject of teaching machines, little space is given to introductory material. After a brief introductory chapter, problems of concern to the equipment design engineer are considered.

The presentation begins by considering the over-all system and then branches into a treatment of each major consideration which the equipment design encounters.

Acknowledgment is given for the assistance of personnel at Entelek, Inc. Of particular importance is their contribution of the results of an analysis of the graphic content of secondary school textbooks in typical use today. Their review of this treatise on engineering considerations was also essential to assure valid perspective and priority of pedagogical values.

D. P. Waite

INFORMATION DYNAMICS CORPORATION
Wakefield, Massachusetts
January 1963

# FOREWORD

Although the Skinner teaching machine is very effective in controlling the conditions of reinforcement in human learning, it confines the stimulus and correct response to a small area, approximately 1" x 7". The student's response must be made in a smaller area. Although minor graphic displays may be shown as part of the item, major graphic and textual displays must be presented in supplements. The paper-backed programmed text introduced after Skinner's prototype machine confines the stimulus and response to even smaller areas. In addition to the loss of graphics, other important features of the textbook are missing in the Skinner-type machine, such as the use of typographical organization to communicate the general outline of the concepts. Chapters, pages, headings, paragraphs, indentations, type face, relative location of illustrations, and so on, are all lost when the linear program slides past the little window of the teaching machine.

The inability of linear programs to take advantage of pattern recognition in learning has also been pointed out by engineers interested in artificial intelligence. In thinking machines, as in man, learning is essential to problem solving. Physical scientists striving to blueprint thinking machines make a distinction between "single thread" solutions, of which their machines have always been capable and heuristic solutions which are only now being investigated. The primary heuristic method sought is to implement pattern recognition.

This report which defines the physical characteristics of teaching devices suitable for the display of graphic media in programmed learning is part of a broader project undertaken for the Northeastern University under a grant from the U.S. Office of Education. The primary purpose of the project was to compare the effectiveness of graphic and symbolic displays of the same concept under the conditions of reinforcement advocated by Skinner and incorporated in his teaching machines. The outcome of this comparison is described in the basic report to which this volume is an appendix.[1]

The viewpoint has been expressed by many individuals in the field that over the long run, a film device will best satisfy the needs for publication and display of the graphic stimuli to be employed in teaching machines. Any method using paper appears too bulky to provide for an extensive library of prepared programs, especially when several sets are required in such user applications as the secondary school system. To escape this difficulty, techniques of miniaturization must be considered.

Several microfilm teaching machines employing optical reduction are already available. The complexities of good photo-optical equipment design, although quite apart from the subject of good programming practice, have a direct bearing on the effectiveness of the over-all auto instruction system. This engineering analysis and definition of the physical characteristics of teaching machines from the engineering point of view will provide a valuable aid to equipment designers contemplating new machine designs. Of greater value, perhaps, will be the service rendered to the community of users and buyers of teaching machines who lack familiarity with the details of the related engineering technology, but who should have an informed basis for evaluating and selecting good machine designs.

A. E. Hickey
Principal Investigator

---

[1] Hickey, A. E., et al. Requirements for graphic teaching machines: Final report on OE Grant No. 7-31-0570-161. Boston: Northeastern Univ., 1962.

## LIST OF ILLUSTRATIONS

FIGURE

1. National Bureau of Standards Resolution Chart
2. Curve Showing Visual Acuity vs. Degrees Eccentric from Fovea
3. Resolving Power as a Function of Image Contrast and Brightness
4. Light Flux Emitted from a Point Source
5. Typical Lens Elements
6. Virtual and Real Objects and Images
7. Converging and Diverging Rays Entering the Eye
8. The Object for Each Surface, After the First, is the Image Formed by the Preceeding Surface
9. Optical Elements in Combination
10. Typical Projector Design
11. An Alternate Collecting Lens System
12. Maximum Illumination of the Object
13. A Virtual Image Projection System
14. Direct Viewing or Screen Viewing
15. Size of Lens Required for Binocular Direct Viewing
16. Minimum Direct Viewing Lens Diameter Effected by Interpupillary Distance
17. Lenz Aberrations
18. Graphics in Music
19. Histogram of Algebra Text Graphic Count by Category
20. Histogram of Biology Text Graphic Count by Category
21. Comparison of Effectiveness of Two Textual Composition Methods
22. Positional Relationships for Viewing
23. Correct Answers vs. Muscular Effort During Learning
24. Calculated Statistics for Students of Secondary School Age
25. Illustration of Effect of Line Length on Reading Speed
26. Typical Projector Bulb Construction
27. Visual Reflectance Spectrum for Manganese Oxide

# CHAPTER I

# INTRODUCTION

The design and development of an over-all system for programmed learning should rest first on sound applied psychology foundations and secondly on sound engineering of the equipment employed. No amount of engineering skill in machine development will accomplish the learning process unless the primary considerations for pedagogical effectiveness in the preparation of the program have been provided.

It is clear, however, that well-prepared program material may suffer in utilization from a machine of marginal or poor design used to present the program to the student. The author has operated new teaching machine models where it was necessary to darken the room in order to achieve suitable contrast on the viewing screen. Other models have been introduced which will not work at all in a darkened room. The fact that the preparation of programs for auto instruction requires professional skills other than engineering suggests that a number of teaching machines of marginal design have been developed on a do-it-yourself basis. The fast-moving pace of the field of teaching machines has encouraged promotion of equipment assembled as adaptations of existing equipment nearest at hand. Although it may give testimony to the effectiveness of auto instruction, the fact that machines of all sorts have been used successfully will give way to a basic set of minimum machine requirements as the field matures.

One of the basic assumptions in this report is allowance for the validity of both linear and branching types of programming. At this time there is insufficient evidence to demonstrate that one of these approaches will win out to the extinction of the other. Since successful programs have been written and demonstrated as effective, the handling of both forms is considered a basic requirement to the teaching machines being considered here.

Something must be said at the outset to provide a common ground of understanding as to what portion of the "teaching machine" field we are interested in. Although the training devices typified by sophisticated military training equipment, for instance, may include the feedback elements of teaching machines, we are not concerned here with such a broad range of applications. Rather, we are limiting the application area and therefore the concept of good machine design to the presentation of programs suitable for use in secondary school systems. It should be clear that criteria for good display equipment design depends a great deal on the nature of graphic information displayed. This point is expanded on later in this report. An effort was made as a starting point to define the representative nature of graphic materials utilized in the secondary school. Although several aspects of the secondary school system application correspond closely, perhaps, with home instruction for adults and children, it should not be assumed that the criteria developed for this application are appropriate in the home, for all industrial applications of auto instruction, or for military applications where skills in physical dexterity or mental alertness are taught.

It is helpful to recognize four major elements of teaching machines.[1]

- ☆ A display which presents the program visually
- ☆ A response panel which the learner uses in forming his response
- ☆ A confirming mechanism which provides the learner with information as to the correctness of his response
- ☆ A reinforcement mechanism which provides the impetus for further operation of the device

Although these are physical aspects of teaching machine configurations, they obviously have implications in the effectiveness of the auto learning process. Unfortunately, a generally accepted theory to explain the learning process is not at hand and we are without benefit of much insight as to the relative effectiveness of alternatives in the design of the last three elements. The importance of proper design in presenting the visual display of the program, however, is so basic that much of value can be said, even in the absence of a generally accepted learning theory.

---

[1] See list of references.

It has been pointed out by workers in the field that the techniques of linear and branch programming expands the consumption of record media over the textbook format. This expansion factor appears so significant as to necessitate some miniaturization of the record media in any practical application where a substantial library of self-instruction programs may be accumulated. This is especially true in secondary school applications where the subject material must be broad in scope and in level of presentation. If one subscribes to the importance of using augmenting graphics to illustrate the text of prepared programs, the need for miniaturization grows even more acute. In the present state of the art, miniaturization, or the storage, retrieval, and display of graphics, suggests either electronic or photographic means.

In the broader field of information handling systems, considerable effort has gone into seeking electronic storage media to enhance compatibility with the capabilities of electronic computers. Within the present state of the art, however, nothing has developed to approach the effectiveness of photographic film media for storage of graphic information. Although active from frame to frame, the information elements of an intrinsic* program cannot be considered dynamic, i.e. as far as the need to change information content recorded on the record media. With an intrinsic program, the ability to change recorded information at electronic speed would serve little or no purpose.

---

* See glossary.

It is argued by some that the full potential of teaching machine concepts will not be realized until the broader capabilities of digital computers are employed to differentiate in more sophisticated ways a greater variety of student responses. Even if it is conceded that additional effectiveness would be achieved through the use of more complex computing equipment, the teaching machine concept is hardly likely to invade the secondary schools if expensive electronic data processing equipment of the high-speed electronic digital computer type is the only approach proposed.

Having concluded from a survey of the literature and a consideration of the soundness of pressures for miniaturization, the reader still faces a widely divergent array of alternate choices of photographic record media and equipment designs for teaching machine use.

Some of the producers of programmed instruction courses appear to have attempted to exploit the broad spectrum of alternatives in equipment design and in the selection of film record media. Some design decisions have obviously been made based on attempts to insure physical uniqueness, hoping that if their programs were not enough to achieve their unique identity, at least the equipment configuration would. This philosophy is shortsighted and self-defeating. It is clear that some degree of standardization must occur in format and selection of film size, for example, as no responsible user organization governing a school system can justify investing in a host of machine designs to gain access to particular programs of their otherwise preferred choice. Sooner than later, a preferred film configuration and a set of equipment minimum performance requirements will be established. There is no more reason to expect a wide disparity in film formats and machine characteristics to endure in teaching machines for the same application than is the case for home movie equipment, for example. There will be more than a single machine concept to meet the requirements of differing applications, but dissimilarities for the sake of uniqueness in compatibility will disappear.

# CHAPTER II

# AUTO INSTRUCTION SYSTEM DESIGN

## Introduction

The teaching machine concepts of interest here are, in the main, general purpose devices. By this is meant that a number of instructional programs independently prepared may be placed in operation using equipment containing the four basic elements of display, response panel, confirming mechanism, and reinforcement device. The prepared programs are selected to fit the subject area or progressive stage of presentation to the student. Otherwise, the system can be identical from student to student. This is the case of interest for application in the secondary school system where a battery of different special purpose teaching machines would be economically unfeasible.

In considering the over-all system, the designer must be familiar with the total problem including:

- ☆ Development of an effective instructional program prepared by experts in the teaching field who have acquired a proficiency and understanding of the teaching machine subject and its particular requirements for material presentation.
- ☆ The detailed design of the equipment used to present the instructional program and to provide for response activities. Once the pedagogical requirements of the machine are well established, this becomes an engineering problem in the selection of components and application of current technology to optimize performance and cost factors.
- ☆ The production and publication of multiple copies of instructional programs. The economic production of duplicate quantities of prepared programs must be given consideration at the time instructional programs are prepared, and even more so at the time equipment design decisions of compatibility are made.

★ Costs. Although an inherent ingredient in each component effort of an auto instruction system design, the total system cost as seen by the secondary school system buyer must be appreciated by the designer. Since true cost is often an evasive issue, this subject warrants thorough consideration if a system design is to be successfully implemented.

These are the main areas of interest kept in focus as this report develops detailed considerations in the area of physical equipment design and in the economic production of multiple copies of prepared programs. It is left to others more proficient and knowledgeable in the fields of applied psychology and education to properly deal with the subject of preparation of high-quality, effective instructional programs.

### Designer's Scope

Although the engineer concerned with teaching machine design need not be expert in the preparation of programs, he must be aware and understand the main points behind the different schools of thought,having impact on program preparation.

As well described elsewhere(2, 3), present thinking endorses two major forms of instructional programming, namely the linear method and the branching method. Since each has adequately demonstrated its effectiveness when applied to specific subjects to be taught, it appears desirable that a teaching machine designed for secondary school use be compatible with both forms of programming.

It is helpful, also, for the equipment designer to understand what is meant by intrinsic and extrinsic programming(4). The intrinsic technique presents a prepared array of graphic images. The sequence of presentation is determined by the results of the student's answer choice as he progresses on from the starting point of the program. Thus, by means of skillful programming, a subject of instruction can be presented in a way which is largely adaptive to the student's progress in correct assimilation of the material.

Extrinsic program is a term used to describe programs that make use of external data and include computations in controlling not only sequence but content of material presented to the student. This technique implies greater equipment and record media flexibility in adapting the material presented to fit the particular requirements of the student. The concept appears to be one of variation in degree of adaptiveness (5), however. This higher degree of self-adaption is obtained by the use of more complicated equipment, such as digital computers working in conjunction with displays and the other major elements of response, confirming, and reinforcement. At the present time, it appears that this teaching machine system concept is best applied to research and investigation of the learning process. It may become a tool for the design of more effective programs and self-instruction systems by providing a better basic understanding of the relationships at play. However, more flexible the extrinsic program concept using the digital computation requirement may be, it appears to the writer that the major increase in cost is not likely to be offset by sufficient increase in learning effectiveness for secondary school use. The objective system designer must, of course, keep up with progress in the field of extrinsic programming as changes in technology and perhaps a better understanding of the learning process itself could modify this present viewpoint.

Another constraint determined by factors normally outside of the designer's preview are the production practices, techniques, materials, and methods available for recording the program on the media to be used. In addition to making the master, one must appreciate the importance of techniques used for economic production of multiple copies.

In selecting the best record media for an auto instruction system design, the designer must face squarely factors in addition to his own technical convictions. Depending on the affiliations of the designer, he may

1·0
1·1
1·25
1·4
2·8
3·15
3·5
4·0
4·5
2·5
2·2
2·0
1·8
1·6
NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU of STANDARDS RESOLUTION CHART

FIG.

be encouraged to elect a media with unique properties or format in an attempt to assure the sale of both programs and machines. Obviously, this could result in rapid obsolescence, if not prevention of initial acceptance of much worthwhile effort,for secondary school system requirements will undoubtedly demand a high degree of compatibility between machines which they purchase and programs of instruction available from several sources needed to cover the wide scope of subject material required for their student body.

Assuming that a suitable selection of record media has been made with sound judgement about the question of compatibility, the designer turns to his central engineering problem of the machine design. The remaining region of choice still becomes blanketed with alternatives, as the scope of teaching machine equipment design contains considerable complexity.

To insure a properly balanced design approach, the concept of a teaching machine as an integral unit has been subdivided into its functional subsystems. The method used to analyse this design problem and to evolve a set of physical requirements suitable for the display of graphic media in programmed learning has required the careful dissection of the functional sub-system into one further level of detailed factors to be considered in the sub-system design. This approach provides a much clearer understanding of the inter-relation and cross-purpose effects of detailed technical factors. A diagram has been helpful both in listing the factors to be considered under each functional sub-system and in showing the inter-connection of these factors throughout the entire device. The mechanism used to present the analytical discussion that follows is through the use of such a design factor connection diagram. In this analysis, two diagrams (fold-outs I and II) have been used--I, to permit laying out details of the preparation of the program media copy, and II, the factors to be considered in the design of the teaching machine. First, however, a review is presented of technical fundamentals on presentation of graphic information to the human eye.

## Fundamentals of Graphic Information Presentation

Several references(6, 7, 8) describe the anatomy, physiology, and psychology of the visual system, or eye-brain combination. It is important that the reader understand some of the basic parameters by which performance of the visual system may be measured in so far as communication of graphic information is concerned. First, it is necessary to define basic terms.

Visual acuity can be defined as the reciprocal of the visual angle in minutes subtended by the smallest discriminable visual detail at the nodal point or fovea of the eye. As a measure of acuity, the resolving power of the eye is said to be its ability to detect small objects and distinguish fine details in a graphic image. This varies widely depending upon the shape of object, color of the image, light reflected from the background, contrast between object and background, duration of the visual stimulus, and the threshold criterion used to determine whether the object is seen or not.

A widely accepted method for measuring resolving power of light sensitive materials is the bar chart, shown in Figure 1. Appendix A gives full description on the use of this chart to measure resolving power. Although primarily intended to serve as an object image for testing photographic film and optical lenses, it is interesting to use this chart to demonstrate the resolving power of the eye.

As a study of the subject will show, no single value can be quoted for resolving power of a normal eye unless a number of conditions are specified in detail. For present purposes, it may be noted that a representative resolving power at comfortable reading distance of 12 to 14 inches is nominally about 6 lines per millimeter (1/mm). At a minimum focusing distance of 6 inches, about 12 1/mm can be resolved. As contrast of the bar chart is reduced, lower resolving power is experienced. (See Glossary of terms

## Fundamentals of Graphic Information Presentation

Several references(6, 7, 8) describe the anatomy, physiology, and psychology of the visual system, or eye-brain combination. It is important that the reader understand some of the basic parameters by which performance of the visual system may be measured in so far as communication of graphic information is concerned. First, it is necessary to define basic terms.

Visual acuity can be defined as the reciprocal of the visual angle in minutes subtended by the smallest discriminable visual detail at the nodal point or fovea of the eye. As a measure of acuity, the resolving power of the eye is said to be its ability to detect small objects and distinguish fine details in a graphic image. This varies widely depending upon the shape of object, color of the image, light reflected from the background, contrast between object and background, duration of the visual stimulus, and the threshold criterion used to determine whether the object is seen or not.

A widely accepted method for measuring resolving power of light sensitive materials is the bar chart, shown in Figure 1. Appendix A gives full description on the use of this chart to measure resolving power. Although primarily intended to serve as an object image for testing photographic film and optical lenses, it is interesting to use this chart to demonstrate the resolving power of the eye.

As a study of the subject will show, no single value can be quoted for resolving power of a normal eye unless a number of conditions are specified in detail. For present purposes, it may be noted that a representative resolving power at comfortable reading distance of 12 to 14 inches is nominally about 6 lines per millimeter (l/mm). At a minimum focusing distance of 6 inches, about 12 l/mm can be resolved. As contrast of the bar chart is reduced, lower resolving power is experienced. (See Glossary of terms for definition of contrast.) Also, when field brightness is decreased, resolving power will decrease. In viewing black and white images, it is noted that with adequate illumination, the illuminate color has negligible effect on the eye's acuity. However, color contrast in an image is an important factor in acuity.

The human eye perceives an image covering a wide field angle of vision. The visual acuity varies over this field, being highest near the center where the image on the retina coincides with the fovea. Figure #2 shows how visual acuity decreases as a function of degrees eccentric from the fovea.

Figure 3 shows in graphic form the relationships between contrast, brightness, and resolving power of great interest to the equipment designer.

Other features of seeing are important in presenting graphic information, They include the three-dimensional properties of seeing, especially the three-dimensional perception which can be created by two two-dimensional views.

Other characteristics of the eye, such as the presence of after images, optical illustions, and the formation of double images are normally to be prevented in order that anamolies not be created in the presentation. Special effects to create apparent motion may be exploited, however, so aptly demonstrated by the moving picture projector.

A basic understanding of characteristics of the visual system is essential to producing a display for optimum presentation of graphic information.

Most displays employ several sub-system elements which affect the over-all system resolution. A relationship for predicting over-all system resolving power when the individual element resolving powers are known is:

$$\frac{1}{R_0^{1.8}} = \frac{1}{R_1^{1.8}} + \frac{1}{R_2^{1.8}} + \frac{1}{R_3^{1.8}}$$

where $R_0$ = output resolution (over-all system resolution)

$R_1$, $R_2$, $R_3$ = resolution capabilities of each image storage or transmission element in the system

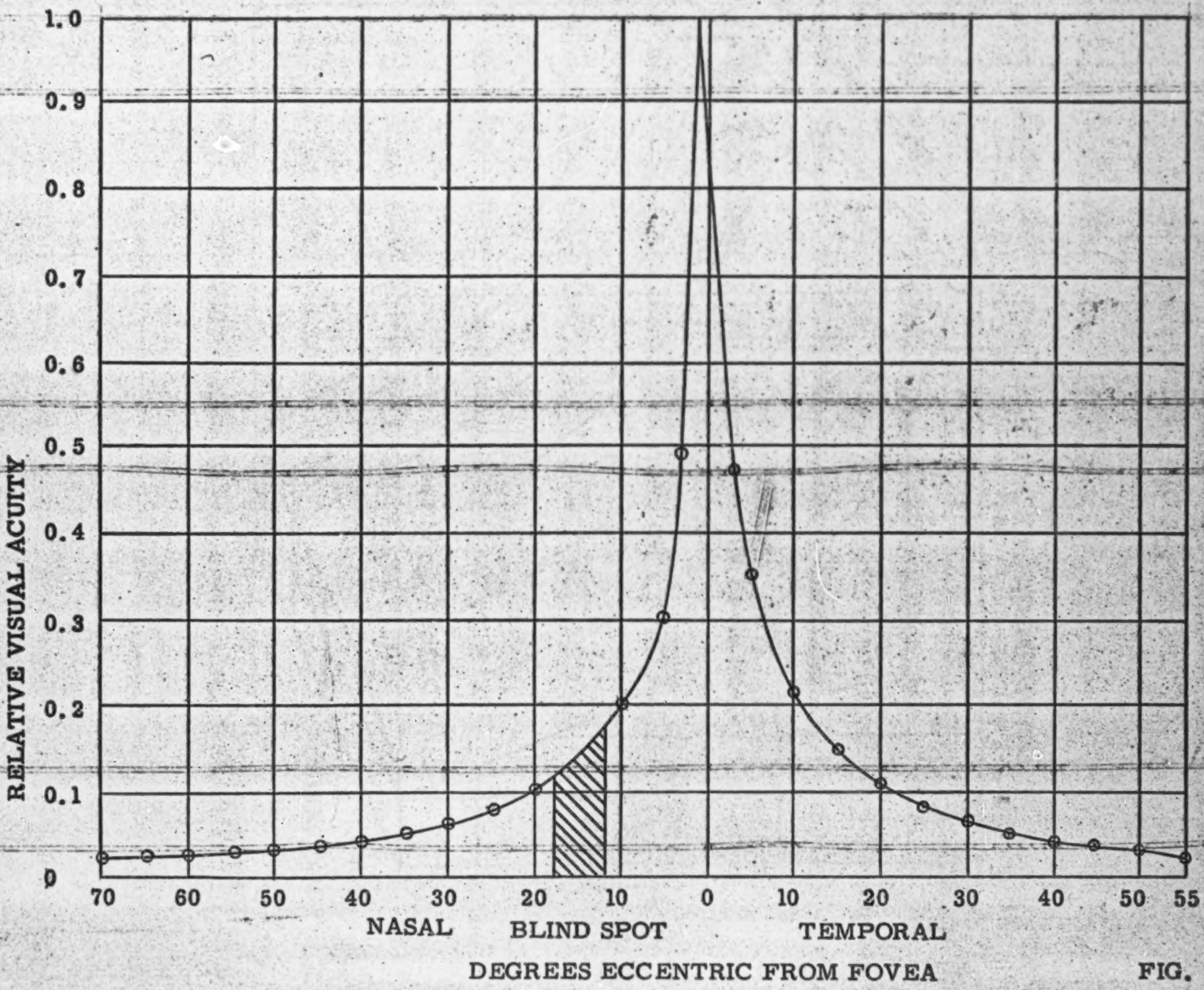

RELATIVE VISUAL ACUITY
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
70
60
50
40
30
20
10
0
10
20
30
40
50
55
NASAL
BLIND SPOT
TEMPORAL
DEGREES ECCENTRIC FROM FOVEA

FIG. 2

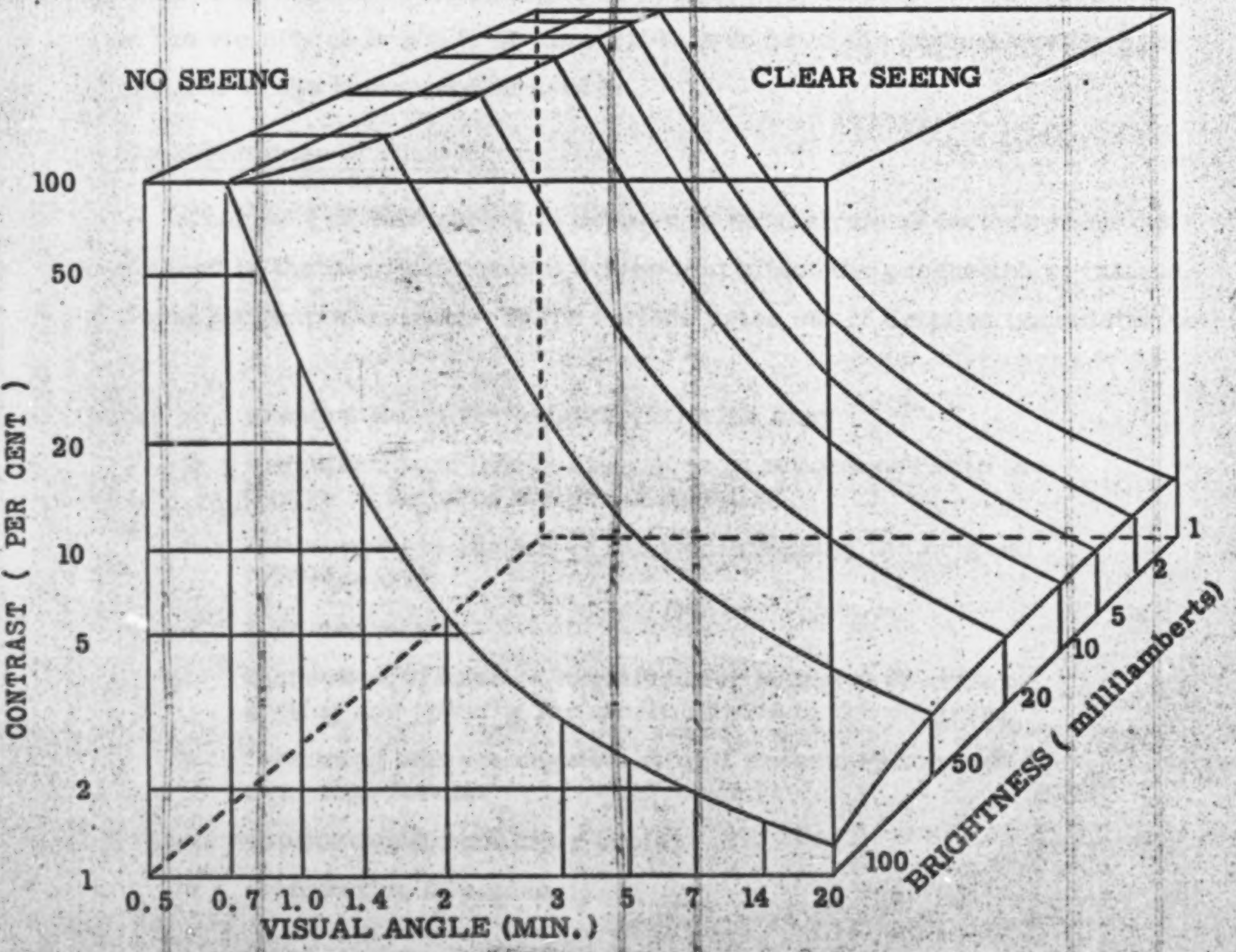

NO SEEING
CLEAR SEEING
100
50
20
10
5
2
1
CONTRAST ( PER CENT )
0.5
0.7
1.0
1.4
2
3
5
7
14
20
VISUAL ANGLE (MIN.)
1
2
5
10
20
50
100
BRIGHTNESS ( millilamberts)

FIG. 3

In a typical case, photographic film containing the program images will have one resolving power. The optical lens system will have another resolving power, and the display screen will have a third. It follows from the above discussion that images as viewed on a display screen should be in the vicinity of 6 l/mm or above if it is to be of the highest quality perceptible to the human visual system.

## Considerations of Film Production

Chapter 7 of this report is devoted to an analysis of factors to be considered in the over-all system design that affect the production of instructional programs on film. These factors arise out of detailed considerations for:

- ☆ preparation of the original full-size copy
- ☆ organization of frame sequences in accordance with the design of the program for instruction
- ☆ initial camera operation for photographing the original full-size copy
- ☆ film processing
- ☆ inspection of camera negatives for removal of errors; editing and splicing for modifications to the program
- ☆ formating camera negative film if necessary to produce user film format
- ☆ duplication for multiple copies
- ☆ distribution of copies

Although the student or user of the teaching machine is not particularly conscious of these aspects of the over-all system design, they are very important technological details as they affect the over-all cost of the system.

## Detailed Considerations of Equipment Design

A detailed subdivision of the equipment design problem showed several functional sub-systems, or design matters, that must be considered. Those included in the detailed analysis of Chapter 6 include:

☆ physical environment conditions

☆ visual effect produced

☆ image projection sub-system

☆ compatibility and accomodation of prepared programs of instruction on film and detailed properties of film

☆ the method for inserting and removing the film from the machine

☆ frame advance and locator mechanisms

☆ non-instructional controls for the operator student

☆ student response provisions

☆ performance evaluator

The problems of the display unit have been expanded for detailed engineering consideration. The other basic elements of response, confirming, and reinforcement functions of the machine design are more narrowly treated. As the state of the art presently stands, knowledge is limited about the alternatives and effectiveness thereof in the design of response, confirming, and reinforcement means. Until more basic information is available, the equipment designer must be content to simply recognize the need for these elements and to integrate his decisions in design selections for them with those for the display design.

Not to be overlooked is the practice of integrating audio channels with the visual display. This report does not include a detailed analysis of this additional dimension of communication in the teaching machine. It is clear that unique problems result from combination of audio and visual presentations simultaneously.

Because of the dominant importance of the projection display in teaching machine design for use of miniaturized program media, a separate chapter is presented to acquaint the reader with projection display engineering fundamentals.

# CHAPTER III

# THE FUNDAMENTALS OF PROJECTION DISPLAY

## Elements of an Optical Display System

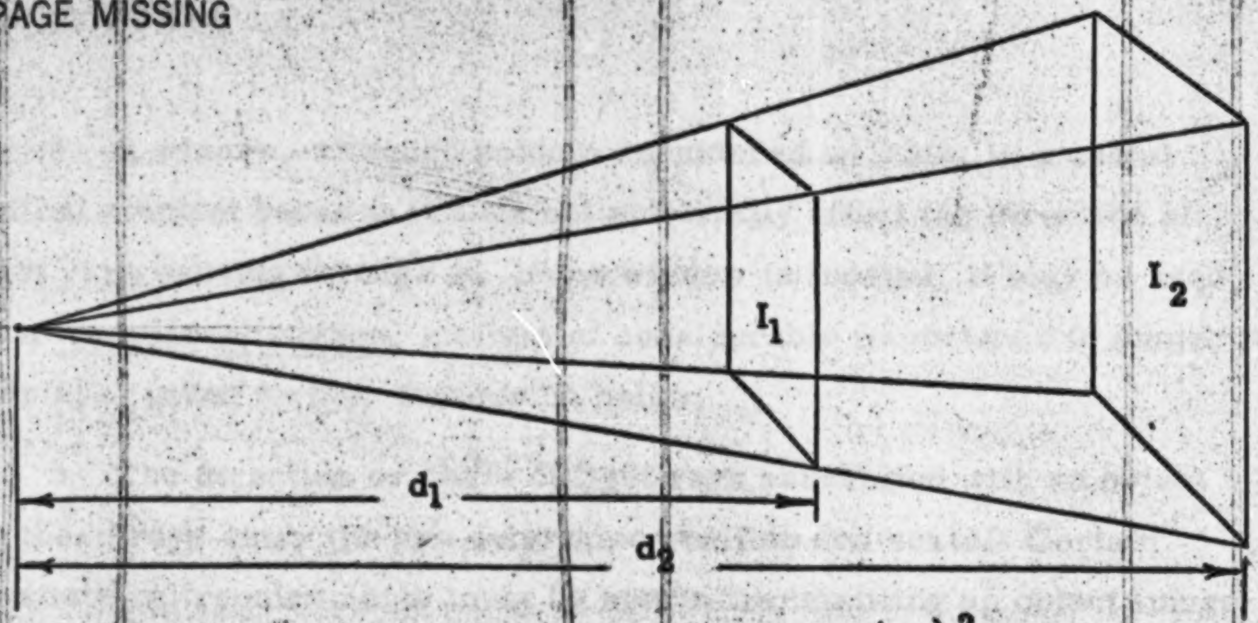
The eye sees an object or its image as a result of the light emitted, transmitted through, or reflected from that object. Without concerning ourselves about the internal workings of the eye, we can review basic elements employed in simple optical display systems. A wide range of design choices rests upon relatively few basic concepts.

1. A point source of light radiates uniformly in all radial directions and is perceived by the eye as a point object. Diverging rays result in a decrease in illumination of any surface exposed if the separation distance to the source is increased according to the well-known inverse square law. This is shown in Figure 4.

2. More complex objects of greater size than the point source can normally be considered as made up of a number of microscopic point sources. The light from each element of the object can be examined as though the element were a point source to better understand the operation of an optical system. This technique, often called ray tracing, falls short when applied to more sophisticated studies in the field of physics, but will, in all cases, be adequate to the equipment designer not concerned with manufacture of the optics themselves but with their application.

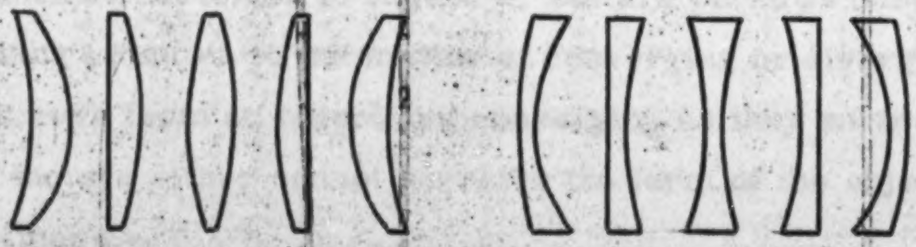
3. Optical materials change the direction of travel of light rays. If the effect of the material's shape and transmission or reflective properties is to change the direction of bundles of rays in certain useful patterned ways,, an optical element results. Thus, the variety of lens shapes illustrated in Figure 5 constitute useful optical elements for their geometry and result in a useful patterned effect on the direction of light rays as they pass through the material.

PRECEDING PAGE MISSING -Fig.4



$$\frac{I_2}{I_1} = \left(\frac{d_1}{d_2}\right)^2$$

FIG. 4


(a) (b)

(a) Meniscus, plano-convex, and double-convex converging lenses.
(b) Meniscus, plano-concave, and double-concave diverging lenses.

FIG. 5

4. A window, although seldom considered as such, is a useful optical element because it does not materially affect the direction of light rays passing through it. If the window is frosted, it may be used as a translucent screen, an item of considerable importance in projection displays, given further discussion below.

5. The direction or angle of light rays associated with an object as these rays enter the eye determine position and scale. Certain geometrical requirements must be met in transmitting an object image to the eye in order that the image or a useful portion of it will be in focus. These requirements are of principal consideration to the manufacturer of optical elements as the conditions for perfect focus can be violated as a result of many causes.

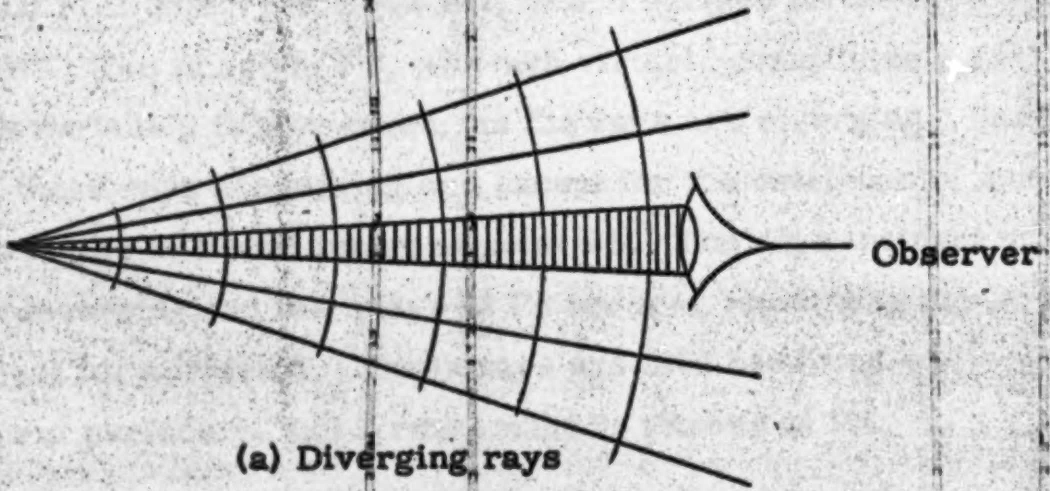
6. Setting aside for a moment the complexities of design of the optical elements themselves, it is important to understand the concepts of real and virtual images. The properties of real and virtual images can be described by mathematical terms in Figure 6, but are perhaps best understood by giving attention to the matter of converging or diverging rays. If the light rays from an object are converging as they enter the eye of Figure 7, the eye either cannot perceive the form of the object or it sees a virtual image.

7. The eye can perceive real images only if the light rays from the object enter the eye in a diverging pattern.

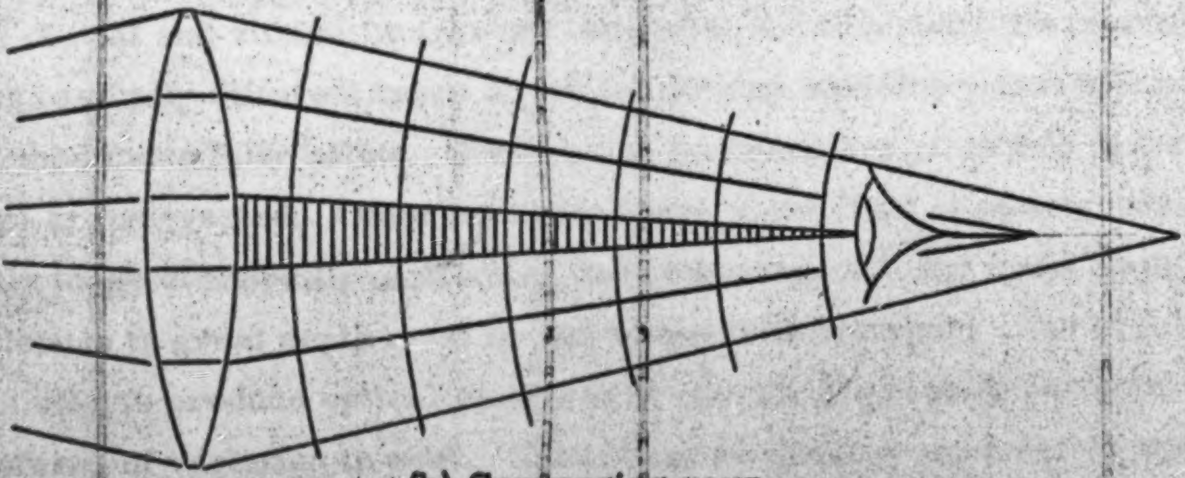
8. The words "object" and "image" have definite significance in the parlance of optical engineers. The light rays we refer to traveling in bundles travel from source to receiver or from object to image. The object is on the source end of the ray and the image at the receiver end. An object in this terminology may not be a physical object at all, but a pattern of light rays passing through a surface which have properties like those emanating from a physical object. If, for instance, two lens elements are

|  |  |  |
|---|---|---|
| Objects | Incident rays diverging | Real object |
|  | Incident rays converging | Virtual object |
|  | Incident rays parallel | Object either real or virtual |
| Images | Refracted rays converging | Real image |
|  | Refracted rays diverging | Virtual image |
|  | Refracted rays parallel | Image either real or virtual |

FIG. 6



(a) Diverging rays



(b) Converging rays

FIG. 7

used in combination, the first lens may have as an object the pattern of light emanating from a physical arrow shown in Figure 8. This lens alone is capable of forming an image with the light rays transmitted and depending upon the characteristics of the two lens elements in combination may affect the direction of travel of these light rays so as to produce a real or virtual object for the second lens.

In Figure 9, several optical elements are shown in combination. A narrow cone of rays diverging from the head of the object is traced through the system. The first surface forms a real image at point P1. The image at P1 serves as a real object for surface 2, which surface forms a virtual image at point P2. The image at P2, although virtual, constitutes a real object as far as surface 3 is concerned, as the rays are diverging. Surface 3 renders these rays converging and except for the existence of surface 4 would converge to a real image at point P3. The rays incident on surface 4 are converging and the image at P3 towards which they converge is a virtual object for surface 4. These rays are then rendered converging after refraction at surface 4, and a real image is formed at P4.

The above introductory explanation serves two purposes: The concepts of real and virtual images are essential to understand the operation of two basically different types of optical display systems which will be described more fully below. Second, the complexities of optical systems design are suggested. Other concepts basic to the field of optics, principally those of aberrations affecting image quality, are far more sophisticated and details in great depth must be left to the lens designer. The results of his efforts produce optical elements of varying degrees of perfection and, therefore, of variation in cost. These may be used in combination successfully by the equipment designer if the performance limitations of the elements themselves are known, and the rules for using elements in combination are adequately understood.

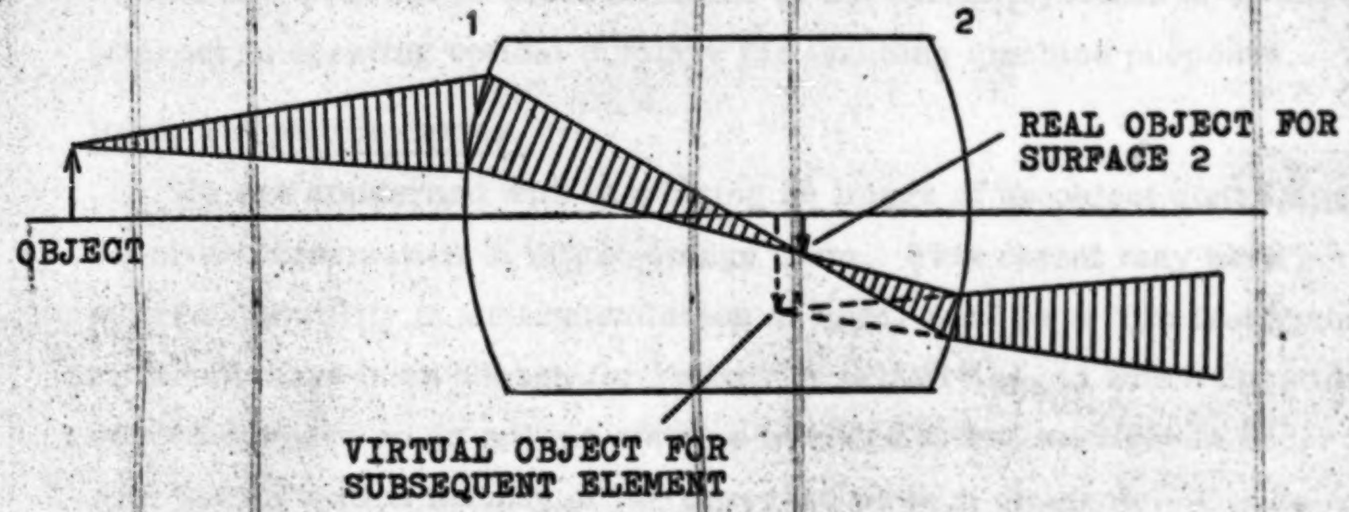
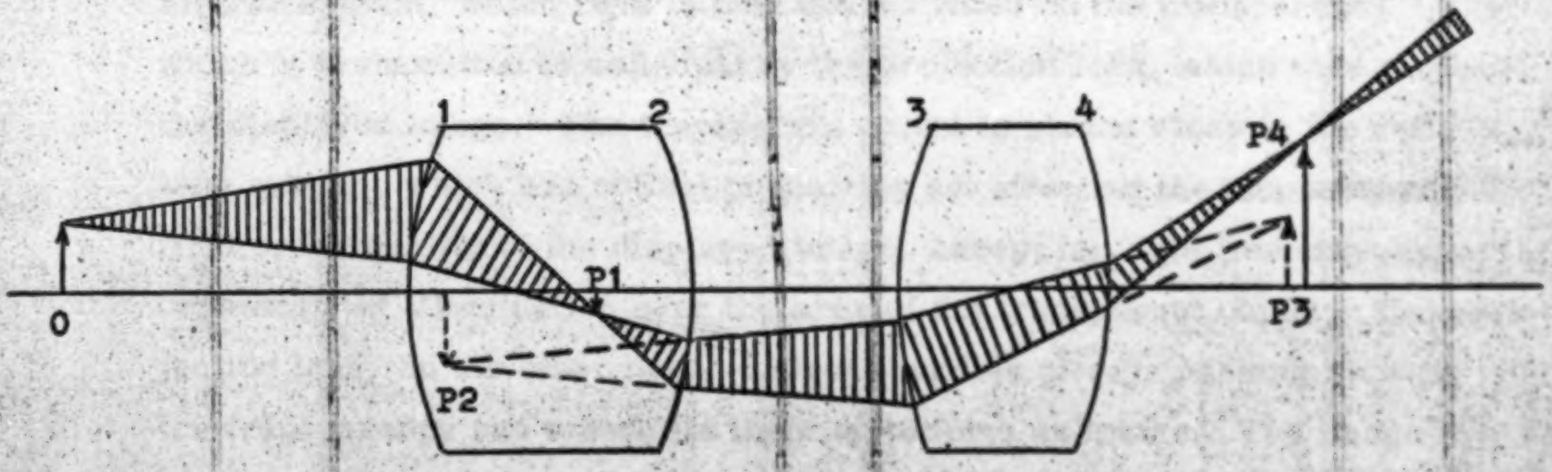

1
2
REAL OBJECT FOR SURFACE 2
OBJECT
VIRTUAL OBJECT FOR SUBSEQUENT ELEMENT

FIG.8


1
2
3
4
P4
P1
0
P3
P2

THE OBJECT FOR EACH SURFACE, AFTER THE FIRST, IS THE IMAGE FORMED BY THE PRECEEDING SURFACE.

FIG. 9

Having presented a brief definition of basic optical element concepts, we can now proceed to describe some of the optical systems of special interest in creating optical displays for teaching machine purposes.

## Basic Projection Optics

We are concerned with displaying an image of an object containing recorded information in micro-image form. This object may have properties of primary utility in its transmission of light, such as a film transparency, or it may have been chosen for reflective properties, in which case the micro record appears on an opaque surface intended to reflect light in accordance with the reflection density of the recorded pattern elements. Optical projection systems to produce images from these two types of objects employ the same principles but are quite different in physical design configuration.

A satisfactory method for projecting images of transparencies employs a high-intensity source of light and a method for collecting up as large a solid cone of light as possible from that source by means of a lens and/or mirror system, which light is then concentrated on the transparency. That which is transmitted is collected by the projection lens, which then produces the displayed image. The transparent object is placed close to the collecting lens system, which has optical properties not affecting the magnification or first order quality of the displayed image, except for light intensity and uniformity of illumination over the area of the transparent object. The projection lens, on the other hand, collects bundles of rays passing through the transparency and transmits them on to form an image. The image may be presented as a display in one of several ways discussed later. We are interested here in the subject of producing an object of sufficient light intensity.

For the projection display of opaque objects, a light source must provide sufficient illumination to the surface area containing the object pattern. In accordance with the detailed content of the pattern, impinging light is then reflected from each element of the pattern. The pattern of reflection also depends on properties of the reflective surface which may be specular, mirror-like or diffuse. Materials with mirror-like properties are not satisfactory for recording patterns for projection as the pattern of reflected light will depend upon the position of the light source, as well as the percentage reflectance of each element in the pattern.

For diffuse materials the intensity of light reflected from each incremental element of the surface is not uniform but varies as a function of the angle which the reflected ray makes with the normal to the surface. For most materials, the reflected light varies as the cosine of this angle. Since each incremental element reflects light in all directions, rather than in only one, as is true for the specular materials, the reflected light can be viewed as though it were eminating from an array of point sources. As long as the pattern is viewed along the normal to diffuse reflecting surface, or nearly so, the image brightness will not vary greatly.

A discussion of detailed considerations in the choice of elements for the display unit is reserved for Chapter 6. A few points of the projector subsystem design present here the fundamentals of the optical systems employed.

In combination with the light source, a reflective mirror is normally used along with condensing lens arranged to accomplish concentration of light emitted from the source on to the object area of the transparency, usually defined by an aperture in the back of the stage or structure which supports the transparency or transparency carrier.

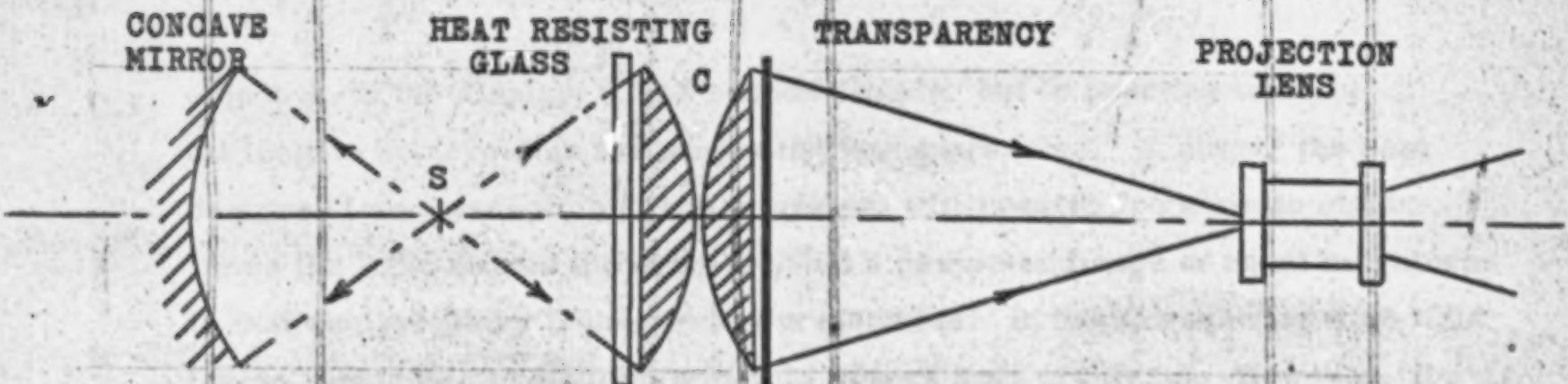
The most common projector design for projecting a transparent object utilizes the arrangement of the optical system as shown in Figure 10. The condenser C collects light from the source S and forms an enlarged image of this of such a size as to just fill the aperture of the projection lens. The latter produces an image of the transparency on the screen. If the source is a coil filament lamp, a spherical mirror may be employed behind the source so arranged that the source is at the center of curvature of the mirror, thus producing a back-reflected image of the source just to one side of the source proper, with a consequent increase in the illuminated area of the source.

The condenser or collecting lens system may consist of two plano convex lenses, as shown in Figure 10, or of a three-lens device, as shown in Figure 11. This later form can be made to reduce the spherical aberrations, and so bring light from all parts of this collecting lens system to a common focus in the plane of the projection lens. The amount of spherical aberration introduced by the condenser may sometimes be too excessive, for if the marginal rays (see Glossary) do not enter the projection lens, a consequent loss of light on the screen will result.

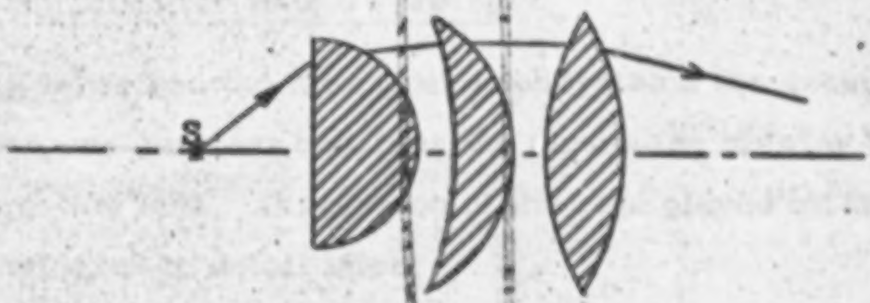
In many cases, it is desirable to introduce a piece of heat-resisting glass between the source and the condenser in order to prevent damage occurring to the projection lens or to the relatively delicate transparency. Modern heat-resisting glass has been found to be very effective in removing the infra-red heat radiation.

An alternate arrangement of optical parts for the illumination of a transparent object when the area is small is to form an image of the source directly on the object, as in Figure 12. This results in greater intensity of illumination of the object, but greater precaution against the damaging effects of heat must be taken. Also, since the rays of light transmitted through the focal point at the plane of the transparency continue to diverge, a larger projection lens is needed to make full use of the light from the transparency. This design approach requires a source having uniform intensity over its whole area. Otherwise, the anomolies are focused in the same plane as the transparency, with the result showing in the display as part of the projected image.

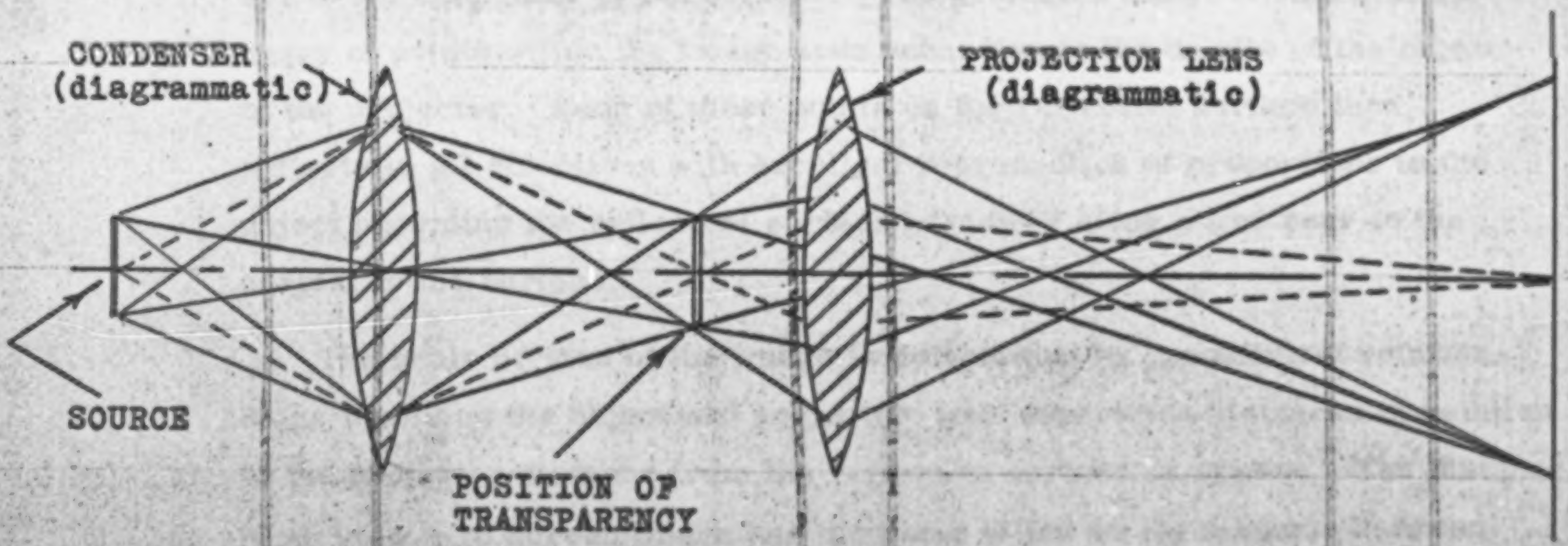
The projection of opaque objects, as indicated above, simply require the illumination of the object so that the projection lens can image the


CONCAVE MIRROR
HEAT RESISTING GLASS
TRANSPARENCY
PROJECTION LENS
C
S

TYPICAL PROJECTOR DESIGN
FIG. 10



THREE LENS COLLECTING SYSTEM
FIG. 11


CONDENSER
(diagrammatic)
PROJECTION LENS
(diagrammatic)
SOURCE
POSITION OF
TRANSPARENCY

MAXIMUM ILLUMINATION OF THE OBJECT
FIG. 12

object on to the display. This sounds simple, but in practice is more difficult to achieve than for projecting transparencies. In short, the best designs of an episcopic projection system will require from six to eight times the light source intensity to yield a projected image of equal brightness to that achievable by transparency projections. In cases requiring high light levels, episcopic projection presents severe heat problems. However, for teaching machine visual displays of desk size, heat would not be the major problem. In addition to heat problems, power consumption must also be recognized as 6 to 8 times greater. This disadvantage is offset in part by the need for fewer optical elements.

## Reflective Surfaces for Image Viewing

Having described the alternative subsystems for generating the object light pattern, we can next consider the means for viewing the image formed by the projection lens. Discussion of the role played by the projection lens will be considered in detail later.

One common method for forming an image is to project on a white or highly reflective surface with both the projector equipment and the viewing eye to the same side of the surface. The projected image illuminates an array of points within the image area according to the details of the object in the projector. Each of these points on the reflective surface then reflects in all directions with excellent reproduction of proportions in the object providing the reflective surface is viewed along a line near to the normal to the surface.

The scale or size of the image is determined by geometrical relationships involving the object and projection lens separation distances in relation to the projector distance from the reflective surface or screen. The distance from viewer to screen image has the same effect as the distance between viewer and any wall mounted picture image.

If the texture of the screen material is apparent to the viewer at the normal viewing distance, this texture pattern will interfere with the image pattern. Many reflective surfaces, however, can be provided with negligible patterns of texture, whether the screen be small, as for a teaching machine, or large, for group displays (for example, movie screen). Special reflective screen materials have been developed for the large group displays as a means of increasing the viewed image brightness for a given illumination level. This is accomplished through the use of specially shaped miniscule elements in the screen surface that do not reflect according to the usual cosine law. Instead, they provide a more directive reflection property, concentrating the reflected light in a cone closer to the normal to the screen surface or to the angle of incident light from the projector (similar to headlight reflectors). Again, the light level problem in teaching machine design is not so great as to require the use of special materials for increasing the brightness of reflected images.

There are disadvantages to the use of reflected surfaces for projection displays. One simple fact is that the projector and the image viewer cannot both be on the normal to the reflective surface, without either vision or the full image projection being obstructed. In large group displays, this is not problem, as the projector is small in comparison to the field of near-normal vision, which can be enjoyed by large number of image viewers. The projector size for a desk-mounted teaching machine, on the other hand, is not practicably reduced beyond a certain size, which is still too large to allow the image viewer to view from the normal to the surface. If the projector is placed off the normal, a distortion occurs in the image, as the geometry is different for the two sides of the image, resulting in truncated pyramid shapes for rectangular proportions in the object. As a result, the image viewer usually views the projected image from an angle of 20 to 30 degrees from the normal to the surface. Although a compromise, this is seldom considered objectionable, as the experience of viewing at even greater angles to the normal is commonplace in everyday reading habits. Although it seems to bother some not to be able to view such an image along the normal, this objection is less pronounced than the objections nearly always raised when viewing translucent screen images as the viewing angle is changed.

## Viewing Translucent Screen Images

If the projector is placed on the opposite side of a translucent screen from the viewing eye, an image can be formed of the object in the projector in the same manner as for a reflective surface, but which is now visible from both sides of the screen. The choice of materials for translucent screens becomes a compromise, for the properties desired work at cross purposes. On one hand, the material should be transparent so that light can pass through the screen to the viewer's eye, yet on the other, transparency cannot be so high as to not produce a diffusing effect on the projected light rays.

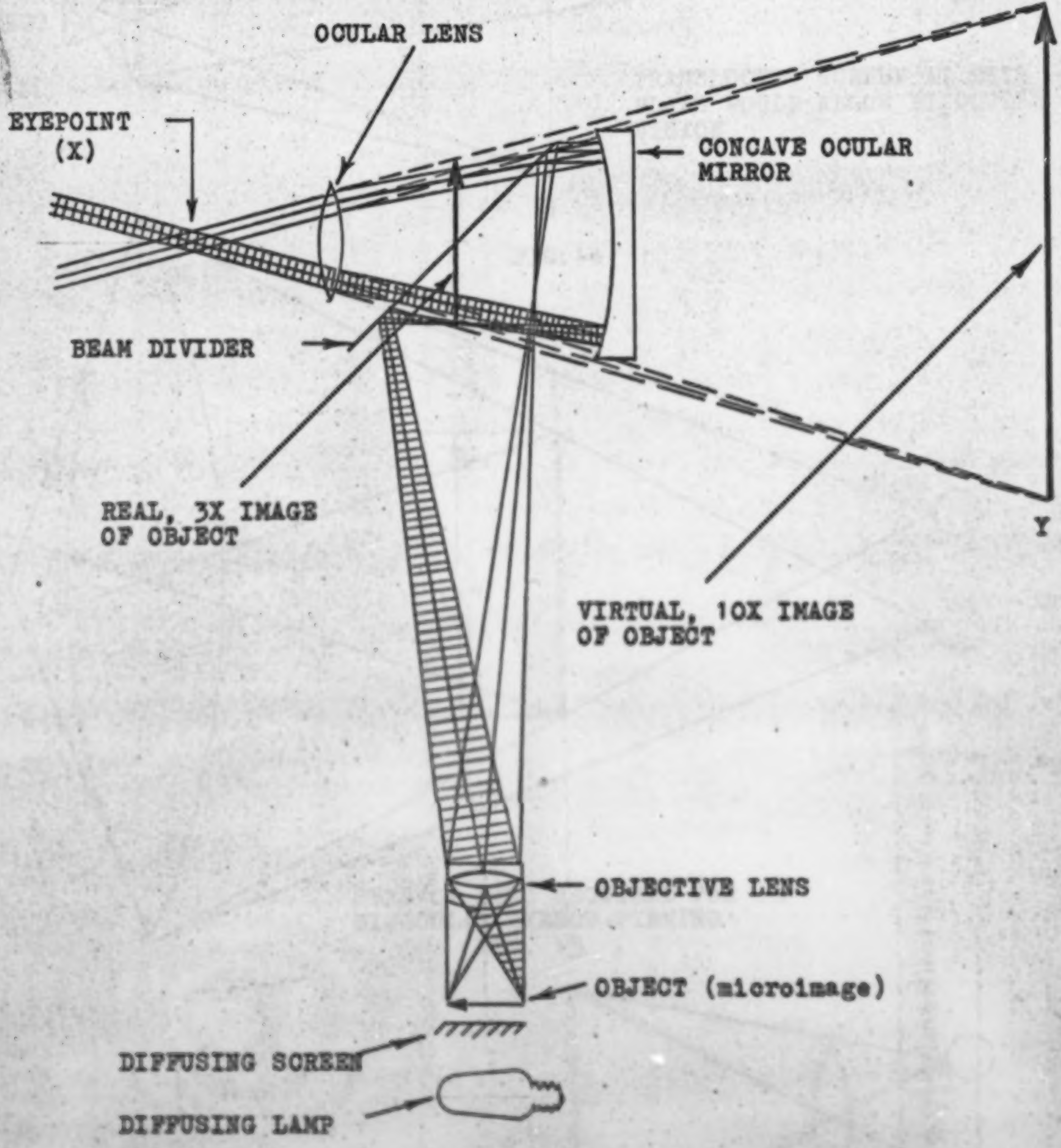
A simple way to understand the operation of a translucent screen is to consider each element of the screen as a dormant point source excited by the impinging light from the projector, which in turn radiates in all directions as like a point source. Materials chosen have, on a microscopic basis, properties that reflect the impinged light in all directions. The preference is for materials which transmit the impinged light to the viewing surface where it is diffused over a wide exit angle, rather than allowed to pass through in a straight or deflected single direction. A perfect translucent screen material would be one which diffused incident light from one side into a wide exit angle with constant uniformity. However, translucent screen materials available fall short of this requirement, with the result that images viewed off the normal to the screen drop off in brightness very quickly. The result is the appearance of a screen image of non-uniform brightness. The degree of this effect is a function of screen size and screen to viewer distance, as well as in the choice of screen material.

## Display by Virtual Images

Images in which the rays of light entering the eye do not actually originate from a real image plane are called virtual images. If the eye is placed at point X in Figure 13, a virtual image will be seen. It appears to be in plane Y. The image seen will be far superior in brightness to that resulting from translucent screen use or from a reflection surface. The increased brightness for a given light source intensity is a result of the large cone angle of light rays in a system which enter the eye when the eye is located at the eye point of the system.

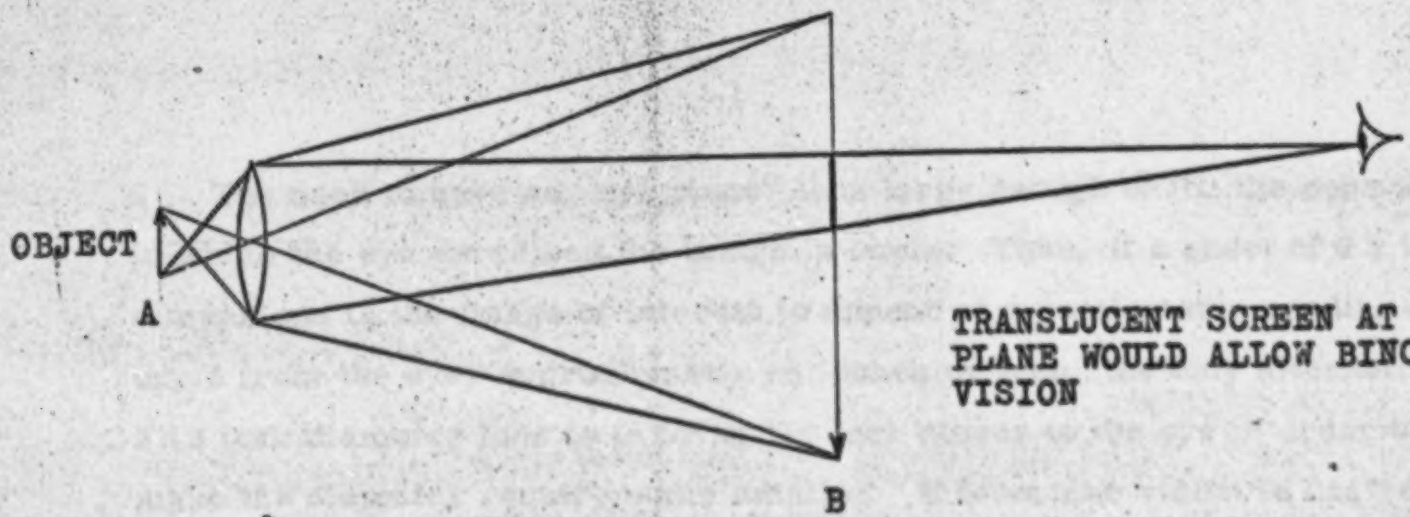
Although the class of projection systems of immediate interest here are virtual image displays, we should for study consider all optical systems in which the eye directly accepts light rays from the optical system without the benefit of an interposed surface, such as reflection or translucent screen materials.

If one considers the simple ocular lens set up of Figure 14, it is seen that conditions are met for providing magnification of an object at point A, producing a magnified image at point B. If a translucent screen were placed at this point, it would emit light in all directions as though it were an array of single point light sources, and binocular vision would be possible with complete freedom of head movement. Without the screen, an eye can be focused on the focal plane (in free space), and the image will be clearly seen. Since no rays from the system other than those transmitted by the lens will enter the eye, only a small portion of the image will be seen by one eye and when viewed binocularly, none of the images seen by both eyes. The result is completely unsatisfactory, and Figure 15 shows what must be done if binocular vision is to be afforded at a distance of ten to twenty inches from the "front surface" end of the machine, with an image size comparable to an 8 1/2 x 11 sheet presented at reading distance. This shows quite clearly that the size of the single lens that would be required is prohibitive both in expense and physical dimensions for compact equipment design.


OCULAR LENS
EYEPOINT (X)
CONCAVE OCULAR MIRROR
BEAM DIVIDER
REAL, 3X IMAGE OF OBJECT
Y
VIRTUAL, 10X IMAGE OF OBJECT
OBJECTIVE LENS
OBJECT (microimage)
DIFFUSING SCREEN
DIFFUSING LAMP

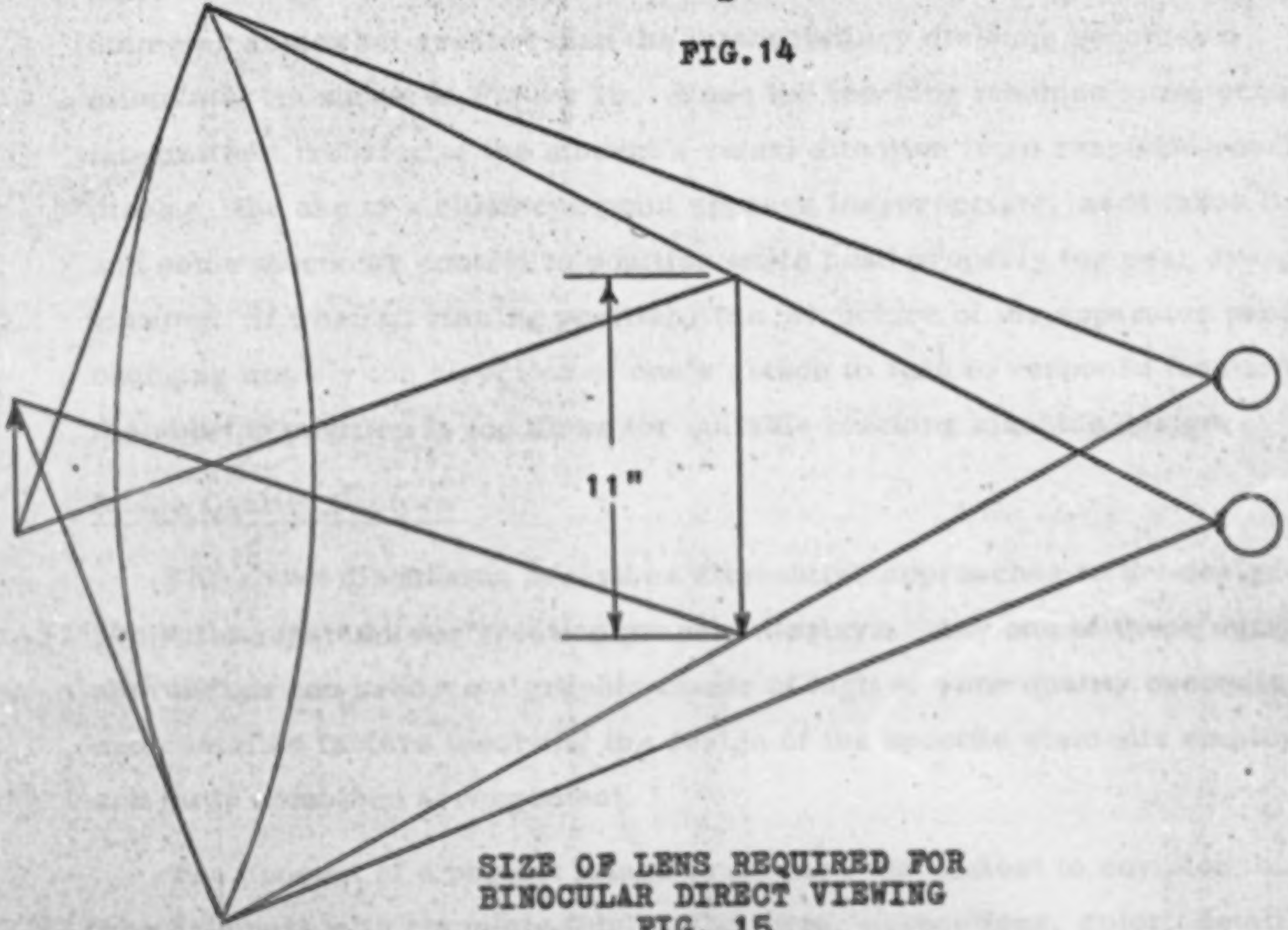
A VIRTUAL IMAGE PROJECTION SYSTEM

FIG. 13

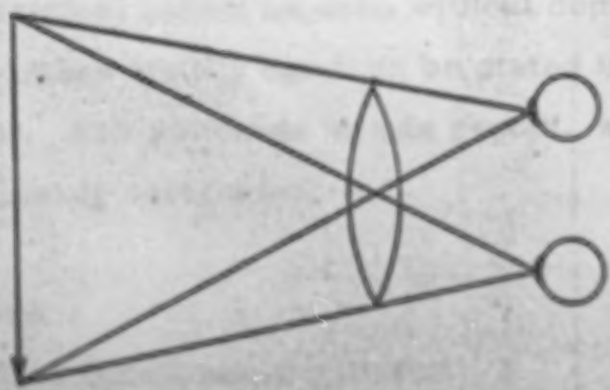


TRANSLUCENT SCREEN AT THIS PLANE WOULD ALLOW BINOCULAR VISION

FIG.14



SIZE OF LENS REQUIRED FOR BINOCULAR DIRECT VIEWING
FIG. 15



MINIMUM DIRECT VIEWING LENS DIAMETER EFFECTED BY THE INTERPUPILLARY DISTANCE

FIG. 16

The need to have an "eye piece" lens large enough to fill the cone angle in which the eye perceives the image is basic. Thus, if a sheet of 9 x 9 inches dimensions is the image of interest to appear at a comfortable reading distance from the eye, approximately 18 inches or less, the only alternative to a 13 inch diameter lens is to bring the lens closer to the eye in order to make the diameter requirements smaller. If binocular vision is desired, a diameter somewhat greater than the interpupillary distance becomes a minimum, as shown in Figure 16. Since the teaching machine's use requires intermittent transfer of the student's visual attention from response panel to display, the use of a close eye point appears inappropriate, as it takes time and some muscular control to position one's head properly for near eye-point viewing. If when in viewing position, the structure of the apparatus prohibits changing merely the direction of one's glance to tend to response functions, the viewing position is too close for suitable teaching machine design.

## Image Quality Factors

The above discussion describes alternative approaches to the design of projection systems for creating graphic displays. Any one of these design approaches can produce a graphic image of high or poor quality depending upon detailed factors involving the design of the specific elements employed and their combined arrangement.

The concept of a perfect image is perhaps the easiest to envision; one which depicts with complete fidelity the form, proportions, color, detail, and all other aspects save scale of the original object as seen without depth but with full depth of focus. Factors of image quality can then be stated in terms of departures from an ideal image. For purposes in this report, we can group these departures into the following categories:

1. Field distortion
2. Lack of sharpness
3. Graininess or other small scale patterns superposed on the image
4. Improper color rendition

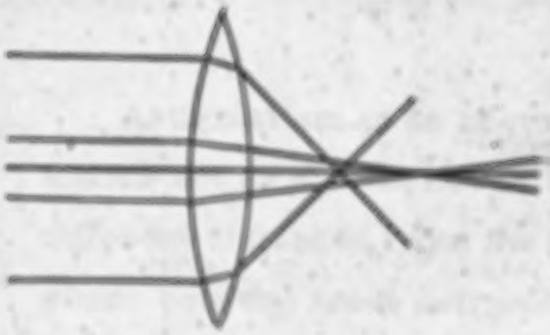
Contrast and brightness are characteristics of an image which are determined principally by the light source and properties of the object pattern. We will include these, however, as additional factors on which we will place criteria to establish an image quality.

An exhaustive discussion of the possible causes of the deterioration of image quality produced by deviations from ideally designed optical elements is inappropriate here. It is desirable, however, to discuss these factors sufficiently to indicate their degree of severity in the teaching machine application.

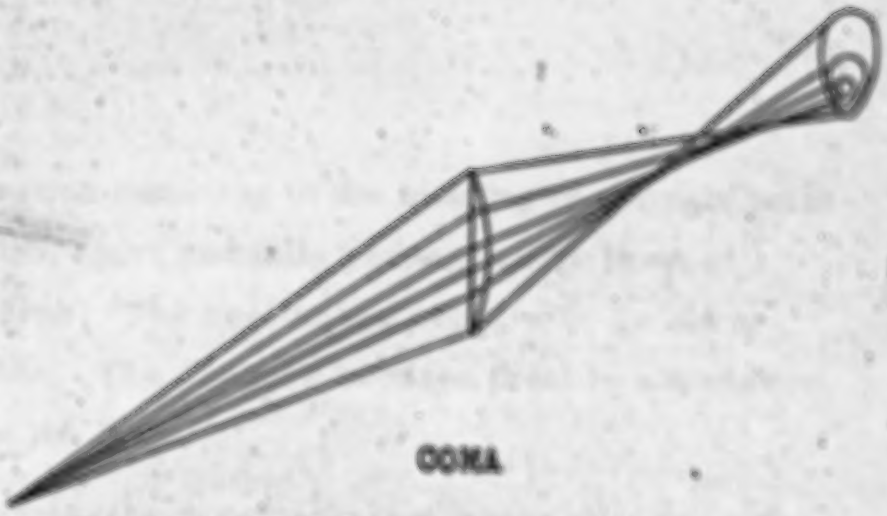
In choosing a lens, the equipment designer must have some familiarity with the aberrations present in projection lenses. The reader is referred to textbooks(8) on this subject for a complete description. It is perhaps sufficient here to indicate in Figure 17 the types of common lens aberrations which should be checked in the selection of a projection lens. It is possible to test a lens by projecting test patterns containing rectangular lines and color patches that will produce noticeable distortions in a projected display if the lens is unsatisfactory for the application.

Spherical aberration must be controlled as its presence directly causes a blurred image, since the light rays emanating from the same object print but passing through different regions of the lens are focused at different points. This type of aberration becomes more difficult to control as focal length is shortened for a given lens aperture.

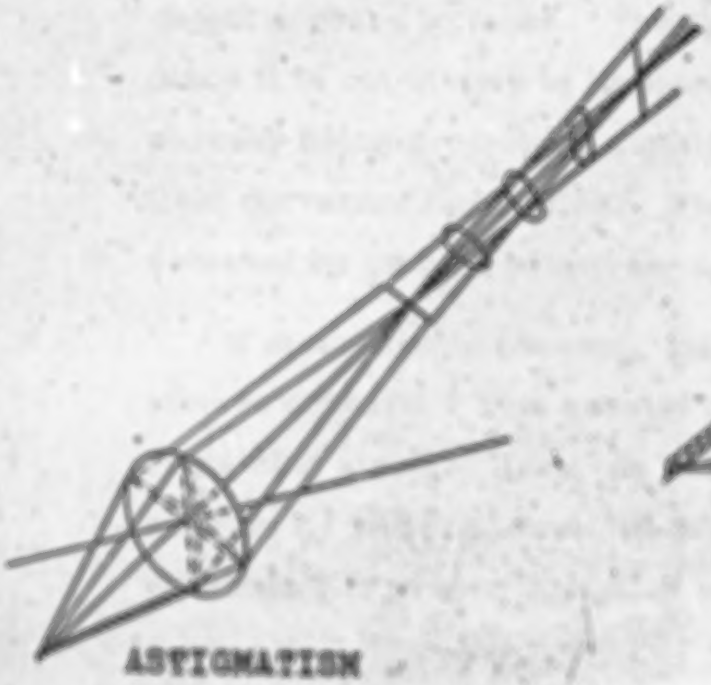
Coma is a kind of lateral spherical aberration. In spherical aberration, the various zones of the lens suffer from a longitudinal difference of focus for rays parallel to the lens axis. Coma affects the rays which enter at an angle to the axis. When coma is present, these oblique rays passing through the various zones converge at different distances from the center of the image so that the image of a single point in the subject roughly resembles an arrow pointing either toward or away from the center of the field.



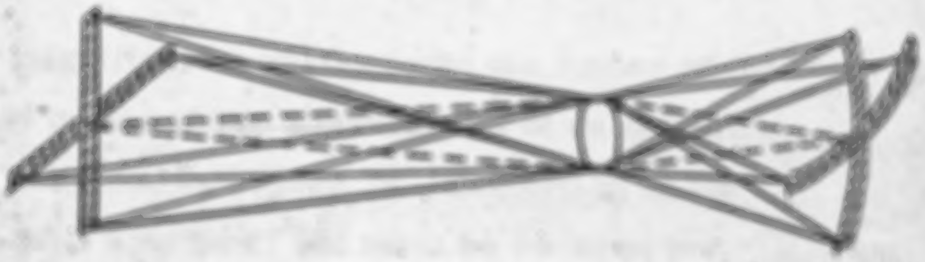
SPHERICAL ABERRATION



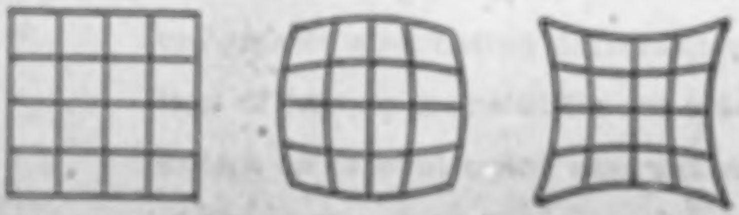
COMA



ASTIGMATISM



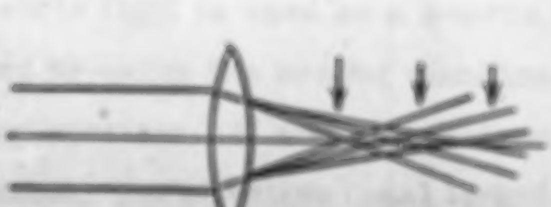
CURVATURE OF FIELD



SUBJECT

DISTORTED IMAGES

DISTORTION



CHROMATIC ABERRATION

LENS ABERRATIONS
FIG. 17

Astigmatism is an aberration resulting in the imaging of a single point source not as a point but as two short mutually perpendicular lines at different distances from the lens. The best image appears to lie somewhere between these two points. The distance between them is a measure of the astigmatism present in the lens.

The field of a lens is the imaginary surface upon which the image of an object appears in focus. The field of a simple lens is not flat but concave. Since it is not simple to produce a compensating curvature in the transparency being projected, a satisfactory lens must employ a means for field curvature correction. Where ample light is present, this is commonly achieved by using a relatively small aperture (center of the lens only).

If distortion is present, magnification varies from the center of the picture outward. This results in one of two appearances of an object containing straight lines, as indicated in Figure 17. Distortion is not reduced by stopping down the lens aperture, but must be reduced or eliminated in proper design of the lens element.

The amount of refraction or bending of light ray of light passing through the lens element varies with the color of light. Thus, the lens acts differently in all of its characteristics for each color of light. Where true color renditions are desired, it is not possible to filter out troublesome colors thereby simplifying the difficulties. Where white light is used as a source, chromatic aberration deficiencies are reduced by using the proper combination of two or more different types of optical materials. If a lens deficiency known as lateral color aberration is also present, the effective focal length of the lens varies from one color to another, thus resulting in differences in magnification of the respective color images. When this is present, colored fringes appear to surround the images in the outer parts of the field. This aberration is not cured by using a small aperture.

The designer may not know or care much about the fine points of optical element design, as this field of technology is so specialized as to necessitate that these matters be left to the experts. It is a simple fact, however, that if the designer will but acquaint himself with the types of difficulties present in optical elements and the means for detecting their presence, he can select without the aid of high-priced laboratory equipment good lenses for the purpose. This is really a necessity, as the suppliers of optical elements provide very low cost lenses to meet the demand for cheap optical entertainment devices and toys. At the same time, the expensive elements used in fine cameras are not required for good projection systems. The designer who selects more knowledgably will accomplish a good quality graphic display image without excessively high cost.

# CHAPTER IV

# MICRO-IMAGE PROGRAM MEDIA CHARACTERISTICS

## Unitized vs. Roll Form

Micro-image media used today in teaching machines include both flat sheet and roll form. Each form has its advantages and disadvantages. It is the objective of this Chapter to describe physical characteristics and other properties of importance of both forms in the teaching machine application.

Questions which must be asked by the teaching machine equipment designer include the following: Should the program be unitized when recorded on the media; that is, should one piece of micro record contain one program of auto instruction? Perhaps it would be advantageous to have a long program subdivided into parts in order to make one copy of the program of simultaneous use to more than one student. If programs are short, but related, perhaps more than one per roll of film has advantages.

The problems of media manipulation by the user in the act of insertion and removal of the program are quite different for a flat sheet in contrast to that of roll form. The mechanisms within the machine design are completely different and present different kinds of problems. It should also be noted that the availability, pricing and cost to reproduce with materials in these two forms are significantly different.

## Roll Form

Although paper comes in roll form and could be used for recording micro images, its wear properties are not suitable for long life and further consideration of this format can be eliminated.

Roll form for micro-images thus reduces to consideration of film. Several sizes of roll film have been established as standards and are available with wide range of emulsion characteristics and even different base materials available in several thicknesses.

Unless roll film is packaged in self-threading cartridges, its use requires a threading operation to fasten the leading end of the film to the transport mechanism. At the same time, it is necessary to mount the reel on a spindle usually powered for rewind. This manipulation is generally conceded to be a disadvantage of roll film. User familiarity with the pass taken by the film through the transport system, if necessary, is bothersome, even though trivial in the mind of the equipment designer. Cartridge packaging and self-threading design removes this concern but may result in significant cost.

Once threaded and in the operating position, the transport mechanism for moving from frame to frame throughout a large collection of images is comparatively simple from a mechanical standpoint. Jumping from a frame at one point in a long roll to a frame far removed is not a rapid process, however, and this point has significance in the construction of branching programs on roll film.

The production of program film duplicates is greatly facilitated if the program is in roll film format, as continuous web materials are easily handled in large volume by production machines. This point, however, may equally apply to flat sheet formats, since flat sheets can be produced originally in continuous web form and then cut into individual units with nearly equal ease if production is high enough.

A very important feature of roll film is the availability of sprocket holes. Other applications of roll form photographic film have necessitated the development and continued supply of several sizes of sprocketed roll film, including 8mm, 16mm, 35mm, 70mm, and even larger film widths. The problem of going from frame to frame of interest in a teaching machine program, as well as proper registration when the frame is found is greatly facilitated if the film incorporates sprocket holes, as this provides a built-in means for metering distance that is precise and non-variant. Most roll film sizes can be obtained without perforations for sprocket holes, with the result that a greater area is available for image recording. In teaching machine applications, this is of minor value, however, and leaves to other means the problem of precise location of individual frames.

Where programs contain several hundred frames, and possibly even thousands within a single program, roll film can offer a single unit of manageable size for the entire program. A one hundred ft. roll of 35mm film fits in a box of 4 x 4 x 2 inches and can easily hold 1500 to 2000 individual frames. Strips of roll film cannot be shorter than 2 or 3 feet and are impractical for most equipment, as leaders are necessary to properly thread film into the transport mechanism. Thus, the subdivision of a program into parts has a practical lower limit when roll film is used. Storage of a large quantity of programs in roll form is not as convenient as the flat sheet form as the film roll wastes space when stored in rectangular containers both because of the existence of a hollow core and the non-conformance to an outer rectangular shape. For most teaching machine users, this point does not appear to be significant.

Film base materials available include clear triacetate normally the least expensive yet quite suitable for most equipment applications. Where high stress is placed on the film, a polyester base can be obtained thereby giving greater tensil strength and longer life to sprocket holes and other points of stress. Use of the more expensive polyester base does not seem necessary in the teaching machine application. Since more than a split second of time is available to go from frame to frame in a program sequence (in contrast to the movie projector), triacetate film of the normal .003 inch thickness basic stock should give long life and have sufficient strength for this application. The thinner base stock results in more compact storage and facilitates controlling film flatness when supported only in tension or under light pressure, as required to maintain focus and minimize image distortion.

## Flat Sheet Form

A flat sheet format makes available use of the lowest cost microrecording materials, namely paper and ink. The Readex Microprint Corporation can duplicate quantities of micro-records on high-gloss paper at reductions of 20 to 1 of line drawings and other textual graphics with surprising fidelity. Since only paper and ink are used to make these cards when produced in quantity, they are the least expensive form of micro-opaque (upwards of 50 copies).

Another micro-opaque form which can be made without special equipment at lower cost in small quantities utilizes photographic paper similar to that used to make photo enlargements. Here, a continuous tone property can be achieved so that pictorial information as well as line copy and text are possible. This feature is not feasible in the printed form of micro-opaque, since the small dot pattern normally used in making half-tones for ink imprinting cannot be controlled for micro-image. The use of opaque cards is feasible as the amount of light needed to illuminate the object, although greater than would be necessary for transparencies, is still achievable without sacrifice in good design, as long as the display is intended for desk-top use with a screen size approximately 9 x 9 inches. For large displays, it is difficult to achieve sufficient screen illumination by the use of opaque objects as discussed in Chapter 3.

A more common form of photographic material used for micro-images is the transparency. As previously described, this enables the use of a less intense light source, yet otherwise provides all of the advantages of photographic record media including continuous gray scale and very high resolution.

Because of the amount of color film work that has been done in the amateur and movie fields, lower cost methods and greater ease of producing duplicate quantities of color film micro-images is perhaps greater than is the case for opaque color micro-copy.

Flat sheet record media is usually easier to handle at time of insertion of the program. The film transport mechanism of the device, however, must then perform more complex operations than in the case of transporting roll film. Mechanical movement in both vertical and horizontal directions are necessary, thereby creating complexity and cost problems if the movement from row to row is to be done automatically. If row to row movement is done manually, this suggests a machine design which requires manual advance from frame to frame in linear or branching sequence, for it seems inadvisable to perform frame to frame advance automatically within rows and yet requires special manual attention to shifting from row to row as the program sequence progresses.

Strong advantages in flat sheet form for micro-images, as far as teaching machine applications are concerned, are not apparent. Although use of this form facilitates subdivision of the program into segments which might be usable simultaneously by different students, breaking the program up into pieces creates problems in keeping elements of the record group together and increases the possibility that loss of an item from a collection of programs can occur thus rendering incomplete a program copy. The flat sheet form has definite advantages in storage, as cited above. However, the number of programs to be stored is not anticipated to be such as to require significant consideration of space savings.

One other factor appears to suggest preference for the use of roll film. The number of frames employed in a programmed course of self-instruction will vary widely as a function of the subject material and level of presentation. If the scope of material to be covered corresponds even closely with that of conventional textbooks of 100 pages or more, the number of frames that can be expected will be at least four to five times this number. This follows in both the linear and branching methods of programming. Unless this total number of frames is subdivided, flat sheet formats for micro-record media would become excessively large unless high reduction ratios are used. Good image quality in the visual display is expensive to achieve if reduction ratios above the range of about 20x or 24x are used. If a 9 x 9 screen size displays one frame at a time, the microform in flat sheet size to carry five hundred pages at a reduction of 24x becomes approximately 80 sq. in., suggesting a possible flat sheet size of 8 1/2 x 11 inches. Such a sheet area requires support from a plastic or cardboard frame and is about the maximum size which affords convenience in handling.

The argument that separation of a program into subdivisions is of great value applies perhaps more to the producer of the program than to the user. If the subdivision is made small enough so that each frame in the program is on a separate record, preparation and subsequent modification of a prepared program is obviously facilitated. Such physical unitizing of each step serves no real value to the user. It is likely to give way to the single record media element capable of containing all frames of the program.

## Use of Standard Sizes

Unless other considerations are so essential as to mitigate against the use of standard sizes resulting from existing applications of suitable film, the designer would do well to choose from existing material formats of standard sizes. This opens the door to the use of standard mechanical parts in the design of the teaching machine itself, as well as assurance that production equipment is available for the preparation of quantities of duplicate copies of prepared programs, and in many cases the existence of services that will process and perform many of the operations necessary to produce duplicate copies for distribution.

The use of 35mm roll film with standard sprocket hole spacing is strongly recommended by the writer. Appendix B contains the ASA standard for this film product. This choice is definitely not made because of its current popularity, but because of a consideration of the elements of choice, one is logically led to this conclusion for secondary school system teaching machine designs. It will be increasingly apparent in the discussion which follows on detail factors of machine design that the use of standard 35mm sprocketed film affords several advantages.

# CHAPTER V

# USER NEEDS IN THE SECONDARY SCHOOLS

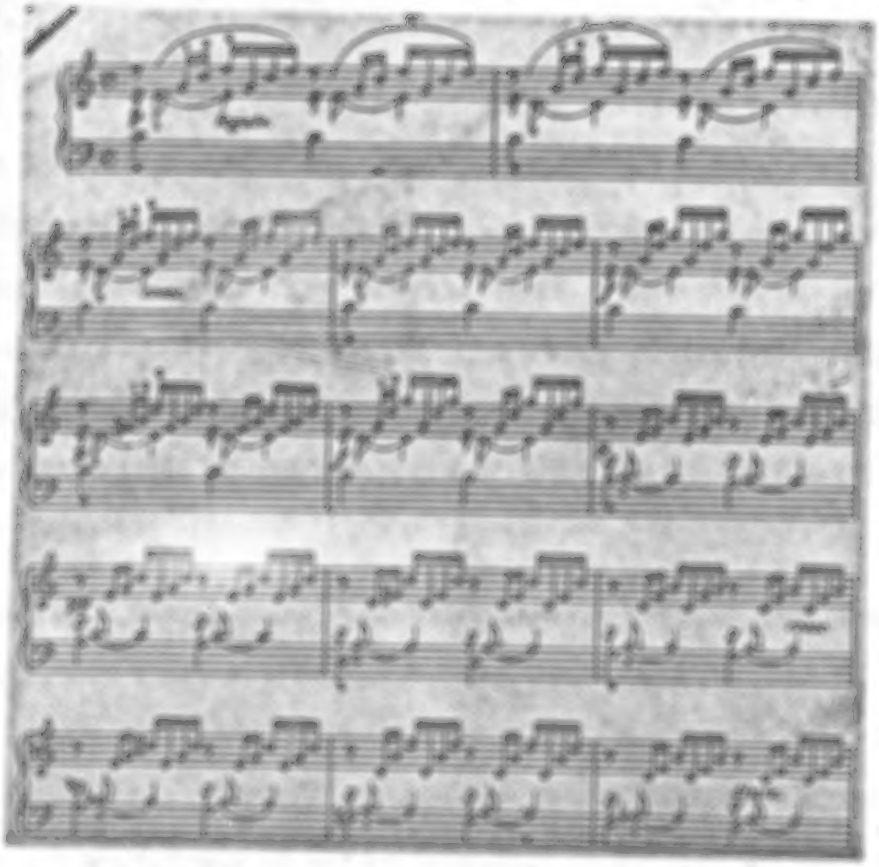
## Graphic Content of Programs

To date, the preparation of programs for auto instruction have not employed the sophistication in graphic presentations that can be ultimately expected. As the field develops and the demand for greater duplicate copies of good programs builds up, the cost of employing more effective graphic presentation methods can be justified. A review of what has happened in the preparation of conventional textbooks, over the last twenty years, for instance, will show a heavy increase in use of color and other graphic reproduction techniques which are more effective.

As a teaching machine equipment designer considers the engineering problems of design in the display unit, it is clear that a description must be given him of the graphic materials to be displayed.

A basic factor determining the nature of graphics in a prepared program is, of course, the nature of the subject matter being presented. A course in music (as in Figure 18) requires a different set of graphic presentations from those of a modern course in biology, not only in the form of graphic information presented, but also in the techniques required to prepare and display those graphics. At the secondary school level a variety of courses are of importance that treat a wide range of subject matter. Thus, an array of graphic spectra requirements exist in the preparation of auto instruction programs for the secondary school simply because of the scope of subject matter to be presented. Add to this the additional fact that progressive courses of instruction usually broaden the scope of materials presented to the student and the range of possible graphic requirements becomes expansive.

It is impossible to precisely define the graphic spectra requirements, as there is no unified agreement at present, nor should there be, for a cookbook form of subject matter presentation. Full reign of continued exercise



The musical staff system is one of the first scientific "graphs" evolved by man. It is essentially a time-frequency-amplitude diagram. The tempered scale which Bach helped develop, actually a logarithmic series with intervals of $^{12}$ 2 , completely revolutionized Western music.

Bach's Preludes and Fugues are beautiful because Bach had the imagination of an artist. They are valid because he employed the disciplines of a scientist.

FIG. 18

of ingenuity in the selection of graphics for effective presentation must be allowed. It is necessary, however, to establish some starting point, though admittedly imperfect, which establishes a gross requirement for graphic presentations at the secondary school level. The approach taken here to establish a minimum framework for the study of graphic display design requirements was based on a sampling of graphics as used in several present-day textbooks in the subjects of algebra and biology. This sampling was done by Entelek, Inc.

The approach taken was to count visual material that was identified as departing from a narrative form, the dominant format of the text. This included pictures, graphs, illustrative algebraic functions, logic diagrams, and tabular material. Details of Entelek's findings are presented in Appendix C. A summary of these findings is presented in Figures 19 and 20. Entelek, Inc. reported that the results measured in these two subject areas were typical of results to be found in the subjects of physics, German, social sciences and music texts. To assist the reader in interpreting titles given to the categories of graphics identified, Appendix D includes copies of several pages typifying these categories. Because the use of color in the reproduction process of this report was impractical, notations substitute.

An examination of Figures 19 and 20 show an interesting comparison. It is not surprising that the use of photographs in presenting materials on biology exceeds the use made of this graphic form in algebra texts. Line graphs are obviously more useful in presenting the concepts of algebra on a percentage basis than is true for the presentation of biology instruction.

Color is used freely in both subjects in present-day textbook reproductions. If the reader has doubts as to the need for this technique in the presentation of textual materials at the secondary school level, he is invited to thumb through not only biology texts, but algebra as well to appraise for himself the increased effectiveness of presentation. This writer believes there is every reason to expect color to be exploited very soon in the preparation of teaching machine programs. This will have a significant effect on the

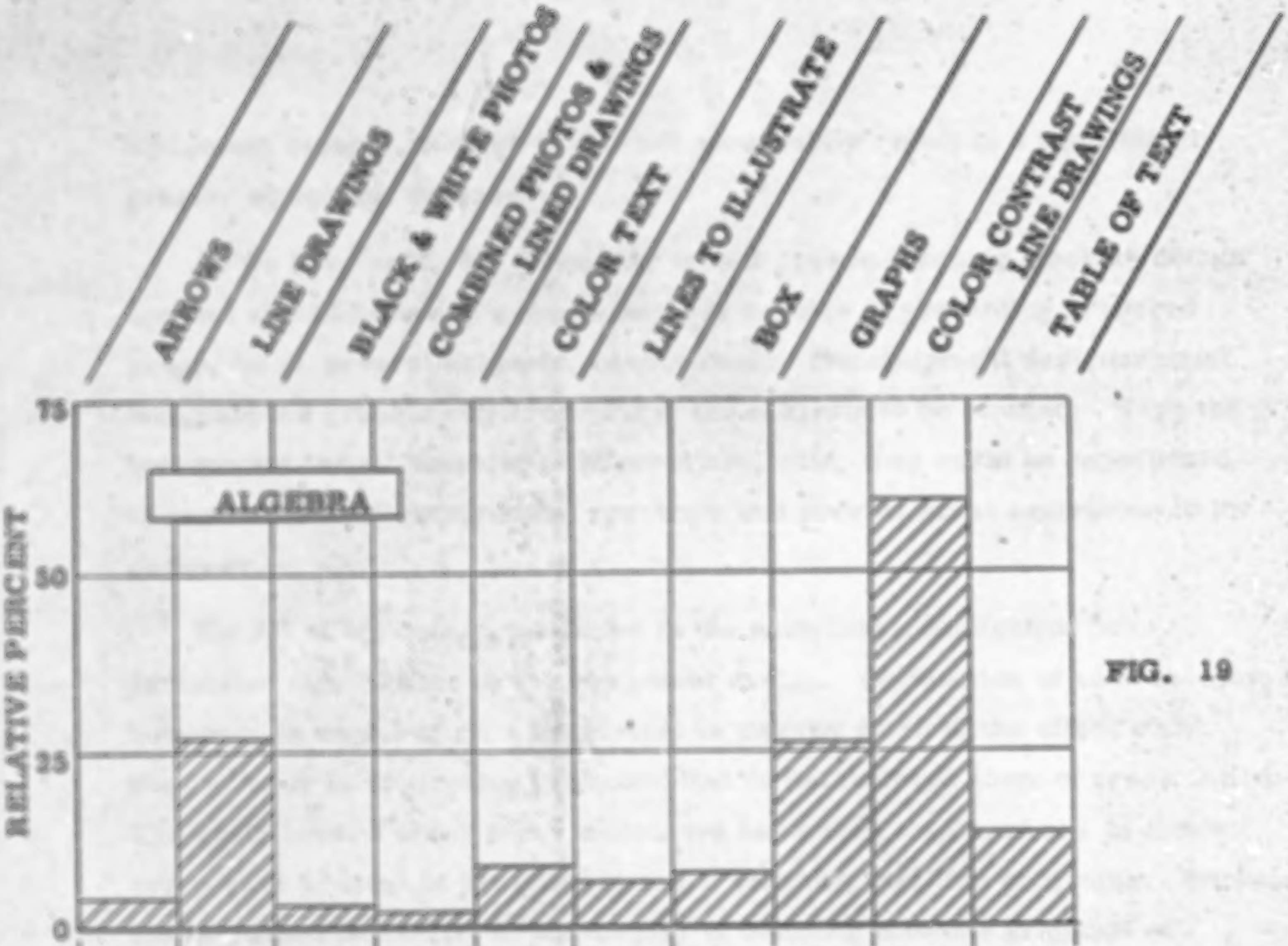


FIG. 19

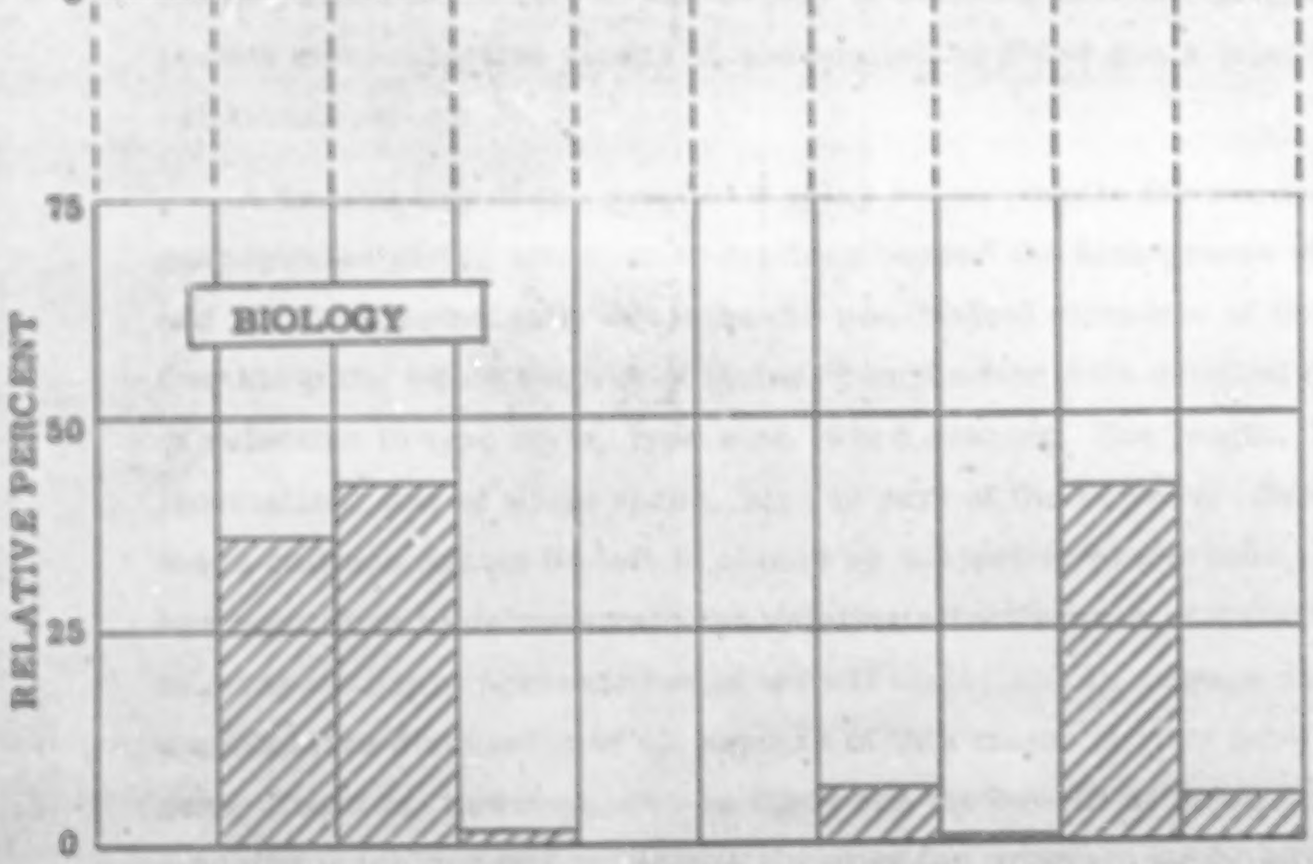


FIG. 20

HISTOGRAMS OF TEXT GRAPHIC COUNT BY CATEGORY equipment design, although it will not necessarily result in a significantly greater equipment expense.

As we have said, the secondary school system teaching machine design concept should be one of general purpose capable of presenting prepared programs on several subjects. Accordingly, the equipment designer must integrate the graphic requirements of the subjects to be handled. Were the histograms for all subjects of interest available, they could be superposed to produce a total requirement spectrum and provide great assistance to the designer.

Not all of the categories listed in the sampling investigation have particular significance in the equipment design. Delineation of each category, however, is useful since a suggestion is thereby given of the effect which the producer is attempting to accomplish in the textbook form of presentation. The goals toward which these techniques have been employed are in almost every case present in the preparation of teaching machine programs. Perhaps the increased flexibility in the display of teaching machine graphics will permit more effective means of accomplishing these goals (clarity, emphasis, relationships, etc.).

A description of the graphic display requirements for secondary education necessitates giving attention to matters beyond the histograms of Figures 19 and 20 which principally describe the non-textual elements of the presentation. Certainly the whole subject of textual composition with detailed considerations of selection in type style, type size, word spacing, line length, line spacing, indentation, use of white space, etc. is part of the picture. Decisions on these matters cannot be left to choice by subjective preference. Good work has been done to demonstrate the relative effectiveness of different approaches to composition in presentation of textual material(10). Space does not permit a qualitative discussion of all aspects of this major subject here. One broad generalization, however, can be made for the benefit of those designing teaching machines and preparing the copy for program publication.

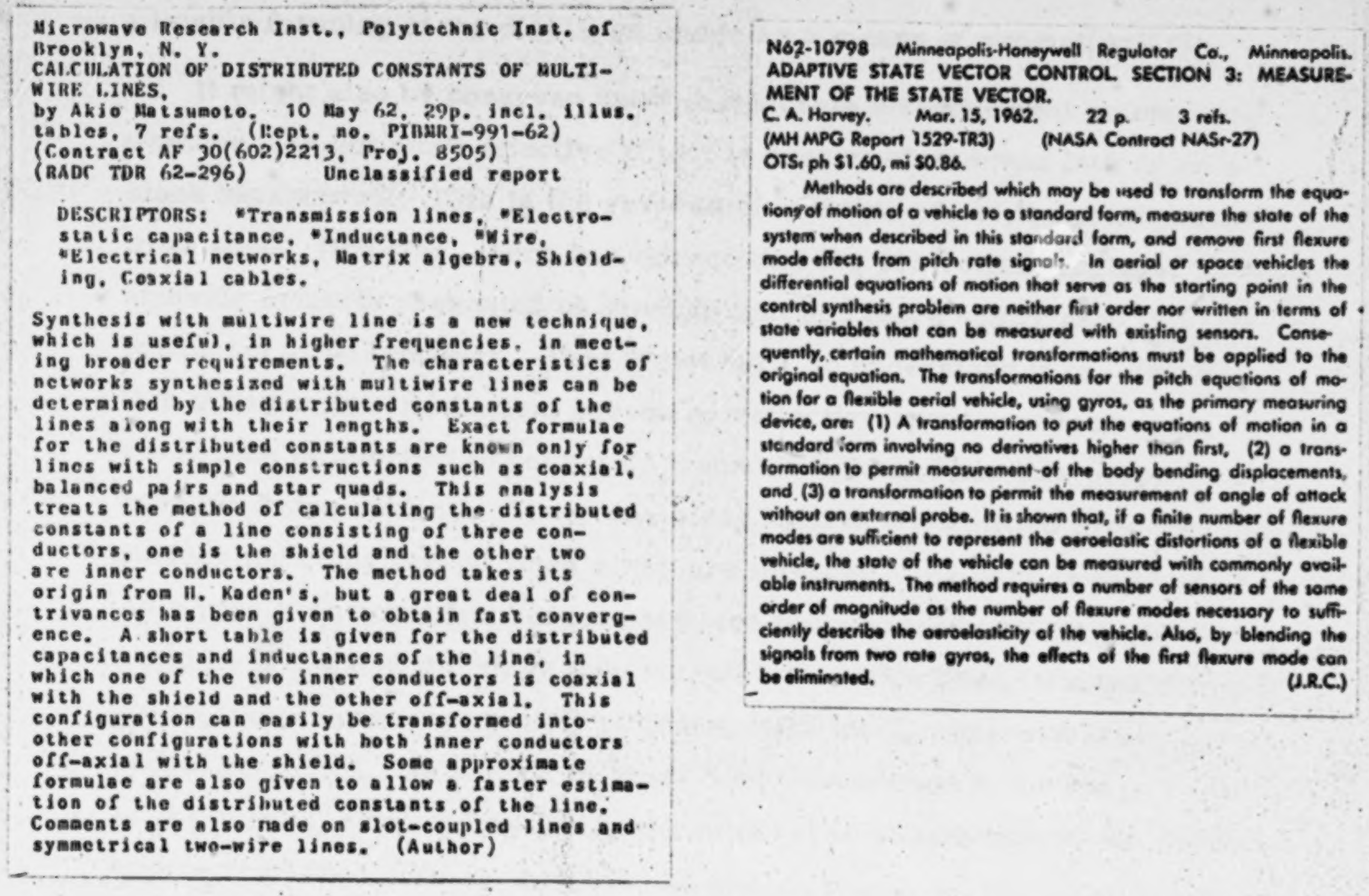
The conventional printed page with associated production methods for recording graphic information as a visual display for the reader's eye has developed into a highly effective medium for graphic communication. Many of the techniques employed in good printing practice have become so commonplace that they are hardly noticed by the layman. In the preparation of programs for auto instruction, the designer of graphic presentations would do well to emulate the achievements of good printing for want of a better guide, until a comprehensive theory of learning is developed to show us more effective techniques.

In composing a program for auto instruction, choice of limited type style and other composition constraints of monospace typewriters as well as no use of color or artistic presentations will result in lower effectiveness and user acceptance of the presentation. The mistake should not be made of concentrating so intently on the values of feedback and reinforcement in the teaching machine concept that sight is lost of the importance of other phychological factors in the creation and presentation of graphic images. The reader will observe in Figure 21a and 21b, the comparison of pure text presentation. These samples were taken from widely used documents prepared by two Government agencies for communication of scientific information in programs on national import. There is little doubt as to the relative effectiveness of these two presentations. Yet the cost to produce these are not significantly different.

## Further Factors in Graphic Presentations

Whereas the printed page and present day practices of printing have contributed a great deal in the development of effective composition means, the printed page is decidedly limited in some ways for use in accomplishing a visual display. If a projection method is employed in a teaching machine design, several new dimensions may be exploited to advantage. One machine concept now marketed which illustrates this point makes use of

2



Microwave Research Inst., Polytechnic Inst. of
Brooklyn, N. Y.
CALCULATION OF DISTRIBUTED CONSTANTS OF MULTI-WIRE LINES,
by Akio Matsumoto. 10 May 62, 29p. incl. illus.
tables, 7 refs. (Rept. no. PIBMRI-991-62)
(Contract AF 30(602)2213, Proj. 8505)
(RADC TDR 62-296) Unclassified report

DESCRIPTORS: *Transmission lines, *Electrostatic capacitance, *Inductance, *Wire, *Electrical networks, Matrix algebra, Shielding, Coaxial cables.

Synthesis with multiwire line is a new technique, which is useful, in higher frequencies, in meeting broader requirements. The characteristics of networks synthesized with multiwire lines can be determined by the distributed constants of the lines along with their lengths. Exact formulae for the distributed constants are known only for lines with simple constructions such as coaxial, balanced pairs and star quads. This analysis treats the method of calculating the distributed constants of a line consisting of three conductors, one is the shield and the other two are inner conductors. The method takes its origin from H. Kaden's, but a great deal of contrivances has been given to obtain fast convergence. A short table is given for the distributed capacitances and inductances of the line, in which one of the two inner conductors is coaxial with the shield and the other off-axial. This configuration can easily be transformed into other configurations with both inner conductors off-axial with the shield. Some approximate formulae are also given to allow a faster estimation of the distributed constants of the line. Comments are also made on slot-coupled lines and symmetrical two-wire lines. (Author)

**N62-10798** Minneapolis-Honeywell Regulator Co., Minneapolis.
**ADAPTIVE STATE VECTOR CONTROL. SECTION 3: MEASUREMENT OF THE STATE VECTOR.**
C. A. Harvey. Mar. 15, 1962. 22 p. 3 refs.
(MH MPG Report 1529-TR3) (NASA Contract NASr-27)
OTS: ph $1.60, mi $0.86.

Methods are described which may be used to transform the equations of motion of a vehicle to a standard form, measure the state of the system when described in this standard form, and remove first flexure mode effects from pitch rate signals. In aerial or space vehicles the differential equations of motion that serve as the starting point in the control synthesis problem are neither first order nor written in terms of state variables that can be measured with existing sensors. Consequently, certain mathematical transformations must be applied to the original equation. The transformations for the pitch equations of motion for a flexible aerial vehicle, using gyros, as the primary measuring device, are: (1) A transformation to put the equations of motion in a standard form involving no derivatives higher than first, (2) a transformation to permit measurement of the body bending displacements, and (3) a transformation to permit the measurement of angle of attack without an external probe. It is shown that, if a finite number of flexure modes are sufficient to represent the aeroelastic distortions of a flexible vehicle, the state of the vehicle can be measured with commonly available instruments. The method requires a number of sensors of the same order of magnitude as the number of flexure modes necessary to sufficiently describe the aeroelasticity of the vehicle. Also, by blending the signals from two rate gyros, the effects of the first flexure mode can be eliminated. (J.R.C.)

COMPARISON OF EFFECTIVENESS OF TWO TEXTUAL COMPOSITION METHODS

FIG. 21

PRECEDING PAGE BLANK-NOT FILMED

variable focusing of the displayed image as a means of prompting(11).

It might also be observed that considerable disagreement exists as to whether it is not more effective if text is presented as white letters on a black background. This is the reverse of common use of the printed page to be sure. However, there is evidence that for projection displays, alpha numeric symbols presented as straight text are more easily read if presented in this manner. This is not hard to understand if one recalls that in either the translucent screen or reflective opaque surface case, the screen is intended to reflect or transmit light and has this property over its entire surface. Thus, any stray light is transmitted over the entire surface including textual lines which are intended to be dark in the image. As a result, the contrast between text and background is not as great as it could be if more of the stray light were masked off. Thus, if a transparent object image consisting mostly of dense, dark background with transparent text is projected, the amount of stray light transmitted in the optical system is diminished, thereby improving the contrast in comparison to the alternate approach where the transparent object passes nearly all the stray light present. Also, it may be argued that the brightness of the textual image displayed can be increased above the ambient level with less discomfort to the viewer as the total amount of light admitted to the eye is far less than if dark characters are presented with a bright background which may make up 90% of the screen area.

The desire to exploit the new dimensions of visual display in a teaching machine design should not be carried too far in one machine. Admittedly, the sophistication of moving picture displays and possibly even three-dimensional displays are possible within the state of the visual aid art and have been incorporated in some teaching machines. However, until such time as these additional dimensions of sophistication are shown to have justifying effectiveness requiring their use as the primary modes of display, the advisability of incorporating these features into a general purpose teaching machine for presentation of the majority of subject matter is doubtful. If relatively few subjects require sophistication in display, it follows that the greater expense of these units will result in fewer units being purchased. The trade-offs should result in availability of the lower cost general purpose machines for more routine material presentations complemented by special purpose machines for presentation of special displays where warranted. The case for designing different machines to create more elaborate displays for special subject materials is genuine and can be appreciated by the buyer. This is where investment in different machines by a school committee makes sense, not in investing in a variety of machines of different design having essentially the same display characteristics but unique only in their detailed construction in such a way as to require that certain program formats go with certain machine design peculiarities.

So far, little has been said about the design of response means, reinforcement techniques, or confirming methods. Unfortunately, so little is known about the relative effectiveness of alternative choices in these areas that little can be said until further investigation is conducted about what is suitable and what is not. There is obviously a great opportunity to incorporate reinforcement and confirming, as well as perhaps even response means, as part of the graphic display. Photographs, especially in color, are a universally appealing form of pleasure, and if subjects are chosen that are not too diversionary, it is easy to see how correct responses can be reinforced by the presentation of pleasing graphics.

Since the principal form of communicating information content of the program to the student is via the graphic display in the machine designs considered here, the confirming information is easily presented as part of the display frame following the student's response. However, confirming information presented in this way may be distracting from the next stimulus element in the presentation. It should not be taken for granted, then, that this method for graphic display of confirming information is optimum. The possibilities of the audio channel providing confirming information to the student's response is attractive, as the spoken work does not remain ever before the student's attention, but is transitory, yet highly communicative, thereby allowing rapid transition of attention to the next subject material being presented.

## Environmental Requirements

During the early days of experimental use of teaching machines in secondary schools it is not surprising to find that separate rooms have been set up for teaching machine installations. The students are asked to leave their regular desk locations and go in small groups to the teaching machine area. Considering the probable impact of the teaching machine, this writer believes that this pilot approach will be transitional in outfitting schools with these tools. Space is a premium in public schools, as every town taxpayer knows, and the best location for teaching machines when in use appears to be the student's regularly assigned desk, providing features of portability, size, and so on are practically considered. If the full potential of teaching machines is to be realized, it appears essential that desk-top units be developed.

In the normal classroom, individual desks are not provided with electrical power. Since installation of such service throughout the school could in itself be expensive, the matter of getting electrical power to the unit is a major consideration. As described in previous chapters, depending on the particular optical system employed for graphic display, power consumption by the light source may be appreciable. In addition, requirements may exist for a powered film transport if full capabilities of the machine design are to be accomplished. Much can be done with manual approaches to advance film from frame to frame. This has been done in one case using flat film cards with considerable ease by incorporating the advance as part of the response mechanism(12). However, the problem of film advance is significantly different between linear and branching programs, and in general, branching programs require greater manipulation of the record media. For roll film, frame skipping is more demanding in power requirement although not necessarily in excess of what can be achieved manually.

The possibility of rechargeable batteries should not be overlooked as the operating power source. The dictating machine used in writing this report will run for an eight-hour day on a nickel cadmium battery, providing completely reliable operation of an electronic amplifier and motor for transporting the magnetic tape. Recharge of the small, dry, encased battery can be accomplished overnight. Though electric cords running down aisles between desks or dropped from the ceiling to each student's machine is unsatisfactory during class operations, such temporary cords for charging batteries when the classroom is not in use would appear satisfactory.

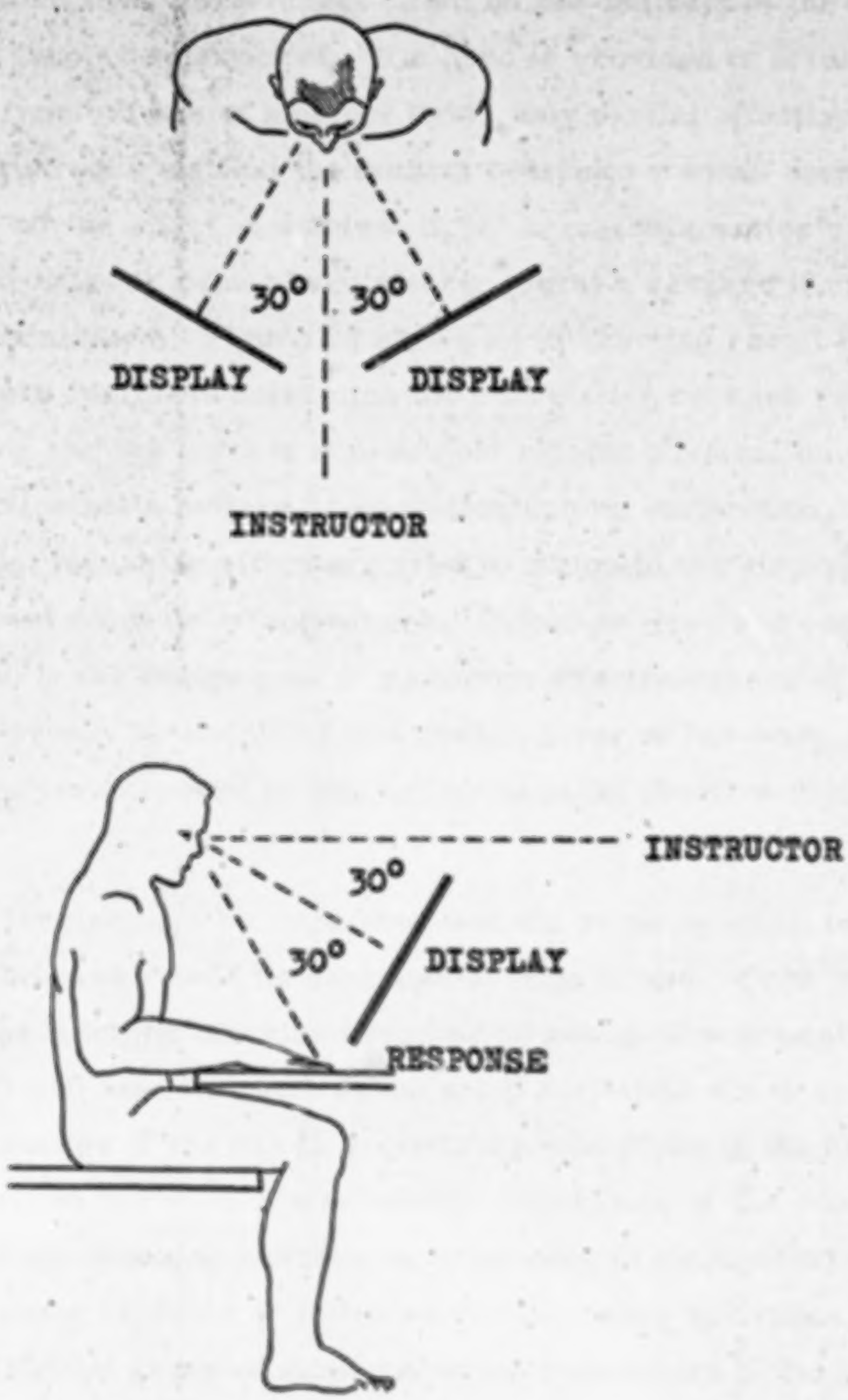
If the teaching machine design includes the feature of portability as well as battery operation, complete freedom for location during use is then afforded without concern for cord length or the proximity of a wall outlet.

Portability is clearly a required element of design for secondary schools. Initially, schools cannot be expected to invest in one machine per student desk and change in machine location must be anticipated. For many purposes during the day, the student's full desk top area is required, thereby necessitating removal of the teaching machine to an intermediate storage area, perhaps at the back of the room, while engaging in other activities. Portability implies both small size and light weight and is enhanced if line cords or other appendages do not exist. Portability also requires rugged construction and an increased concern for safety in the selection of materials, as movement of equipment increases the possibilities for injury to students in the classroom and damage to the equipment.

If the teaching machine becomes a general purpose machine suitable for the presentation of many subject materials, it will enjoy a heavy use factor. This will necessitate the use of components with specifications for continuous rating and long life. The facilities employed in the average secondary school undergo a terrific beating. Pride of ownership is seldom present. The social background, habits, and training of students vary widely. The vandalism trait present in some small percentage of teenagers cannot be disregarded. The range of materials available for construction of electromechanical equipment offers assurance, however, that a fool-proof nondestructible design can be approached. The reader's attention is drawn to the vast improvement in children's toys, both in durability and the harmless feature in designs now being achieved through the use of rubber-like plastics.

Attention needs to be given to the positional relationship between the graphic display surface viewed by the student and the normal seating position of the student. Although it is impractical to achieve complete flexibility in positioning such as can be acheived for the student in reading from an open book, certain fundamental requirements should be met. Figure 22 illustrates these basic requirements. The student should not have to slouch down in his seat to acquire optimum head position for viewing the image. The image surface area should not be at an angle which results in reflections or poor contrast from ambient lights or windows. The machine should not obstruct the normal view between students and instructor at the front of the room. A normal comfortable reading distance between the student and the image being viewed should be provided while at the same time maintaining sufficient clearance between the physical structure of the machine and the student's head to allow simple eye movements to accomplish a switch in attention from the displayed image to the desk top or a distant point of focus at the front of the school room.

Although confining, these requirements do not eliminate the possibilities of effective graphic displays. It does necessitate, however, careful design--even ingenius design--if low cost is to be achieved. Low cost is unquestionably a factor of considerable significance in competing for the secondary school market.


30°
30°
DISPLAY
DISPLAY
INSTRUCTOR
INSTRUCTOR
30°
30°
DISPLAY
RESPONSE

POSITIONAL RELATIONSHIPS FOR VIEWING

FIG. 22

A major consideration in environmental conditions is that of ambient light level. Ambient light has a direct effect on the contrast of the displayed image in several display approaches. If a hood is provided to shield the projected image from effects of ambient light, only partial effectiveness is achieved. Designs requiring that the student peer into a small opening, thereby blocking off the effect of ambient light, are in this writer's view too great a compromise in ease of use and represent a hazzard if clearance distance is not maintained. Figure 23 shows an interesting result from tests that have been run(13) to determine the correlation between retention of pattern learning and the level of non-subject related physical muscular force required to maintain pattern presentation during instruction. It would appear that special muscular effort required to maintain the display in view will directly detract from its effectiveness. Complete ease and convenience in viewing is clearly the design goal if maximum effectiveneness of the display is to be achieved. To sacrifice this goal in favor of low cost, portability, or other factors appears to this writer to be an abortive design approach.

Nor can the requirement be stipulated that the room in which teaching machines are to be used should be darkened at time of use. Once the novelty has passed and the teaching machine becomes an accepted practical tool of the classroom, it will take its place in the many activities which are changed about during the course of the day in accordance with plans of the class instructor, as well as the student's schedule. Darkening of the room creates other problems if the teaching machine is to be used in conjunction with other activities by the same students or at the same time other activities are being conducted by a different group of students or the instructor. If the teaching machine is to make more time available for direct contact between an instructor and the individual student, it cannot simultaneously require they sit together in the dark.

Since image contrast is a function of ambient light level and the latter cannot be controlled, it follows that a control should be provided enabling the student to adjust image brightness level accordingly.

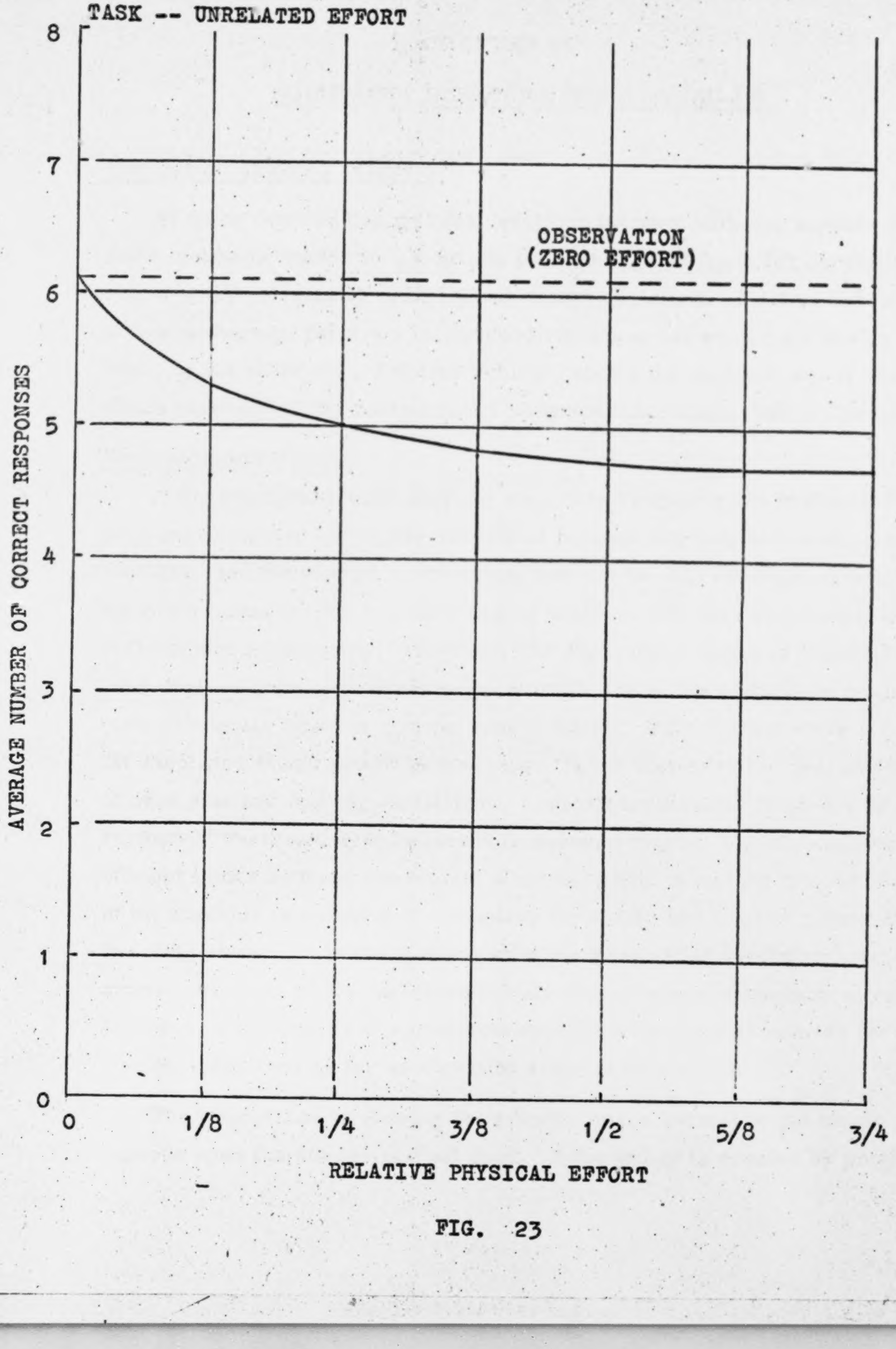

TASK -- UNRELATED EFFORT
OBSERVATION (ZERO EFFORT)
AVERAGE NUMBER OF CORRECT RESPONSES
0
1
2
3
4
5
6
7
8
0
1/8
1/4
3/8
1/2
5/8
3/4
RELATIVE PHYSICAL EFFORT

FIG. 23

PRECEDING PAGE BLANK-NOT FILMED

# CHAPTER VI

# EQUIPMENT DESIGN FACTORS DETAILED

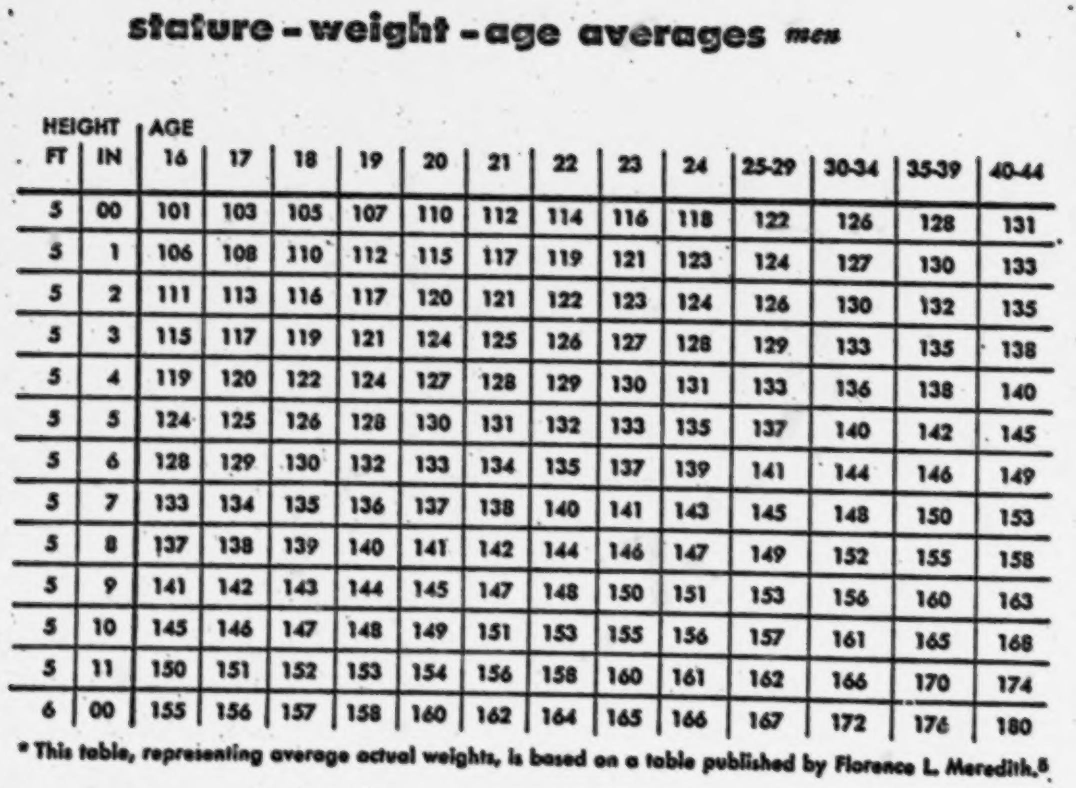
## The Design Analysis Diagram

So many detailed design considerations interact with one another that some means is needed to lay out the whole design problem for careful analysis and over-all integration of individual design decisions. Foldout Exhibit II shows in diagram form ten major subdivisions of the equipment design problem. In the same way, Foldout Exhibit I shows the major areas of production effort essential to the publication of program films discussed in Chapter 7.

## Environmental Factors

If the requirement for desk top size is to be met, a set of dimensions must be developed for proper positional relationship between desk, teaching machine, and the student. The author has not be able to obtain authoritative source information for standard school desks or for body dimensions of children and adolescents. However, the dimensions shown in Figure 24 have been derived from authoritative statistics(6) for adults and can be scaled down proportionately based on age vs. height tables. For comfort while viewing, the displayed image should be no closer than 10 inches to the eye, and because of desk size and spacing limitations, a maximum distance from eye to front surface of the machine of 24 inches is recommended. A minimum clearance of eight inches between the student's normal position and the nearest element of the machine is considered necessary for safety and to permit ease of vision to points elsewhere in the classroom without requiring excessive head movements. Because of the variation in body dimensions of secondary school students, some degree of adjustment appears necessary to provide for optimum viewing conditions as far as elevation angle is concerned.

The importance of viewing the graphic image normal to the image plane depends upon the display method used. If the image is created by projection



**stature - weight - age averages** *men*

| HEIGHT FT | HEIGHT IN | AGE 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25-29 | 30-34 | 35-39 | 40-44 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 00 | 101 | 103 | 105 | 107 | 110 | 112 | 114 | 116 | 118 | 122 | 126 | 128 | 131 |
| 5 | 1 | 106 | 108 | 110 | 112 | 115 | 117 | 119 | 121 | 123 | 124 | 127 | 130 | 133 |
| 5 | 2 | 111 | 113 | 116 | 117 | 120 | 121 | 122 | 123 | 124 | 126 | 130 | 132 | 135 |
| 5 | 3 | 115 | 117 | 119 | 121 | 124 | 125 | 126 | 127 | 128 | 129 | 133 | 135 | 138 |
| 5 | 4 | 119 | 120 | 122 | 124 | 127 | 128 | 129 | 130 | 131 | 133 | 136 | 138 | 140 |
| 5 | 5 | 124 | 125 | 126 | 128 | 130 | 131 | 132 | 133 | 135 | 137 | 140 | 142 | 145 |
| 5 | 6 | 128 | 129 | 130 | 132 | 133 | 134 | 135 | 137 | 139 | 141 | 144 | 146 | 149 |
| 5 | 7 | 133 | 134 | 135 | 136 | 137 | 138 | 140 | 141 | 143 | 145 | 148 | 150 | 153 |
| 5 | 8 | 137 | 138 | 139 | 140 | 141 | 142 | 144 | 146 | 147 | 149 | 152 | 155 | 158 |
| 5 | 9 | 141 | 142 | 143 | 144 | 145 | 147 | 148 | 150 | 151 | 153 | 156 | 160 | 163 |
| 5 | 10 | 145 | 146 | 147 | 148 | 149 | 151 | 153 | 155 | 156 | 157 | 161 | 165 | 168 |
| 5 | 11 | 150 | 151 | 152 | 153 | 154 | 156 | 158 | 160 | 161 | 162 | 166 | 170 | 174 |
| 6 | 00 | 155 | 156 | 157 | 158 | 160 | 162 | 164 | 165 | 166 | 167 | 172 | 176 | 180 |

*** This table, representing average actual weights, is based on a table published by Florence L. Meredith.[5]**

FIG. 24(a)

ANTHROPOMETRIC NOMOGRAPH OF ARMY MEN[a]

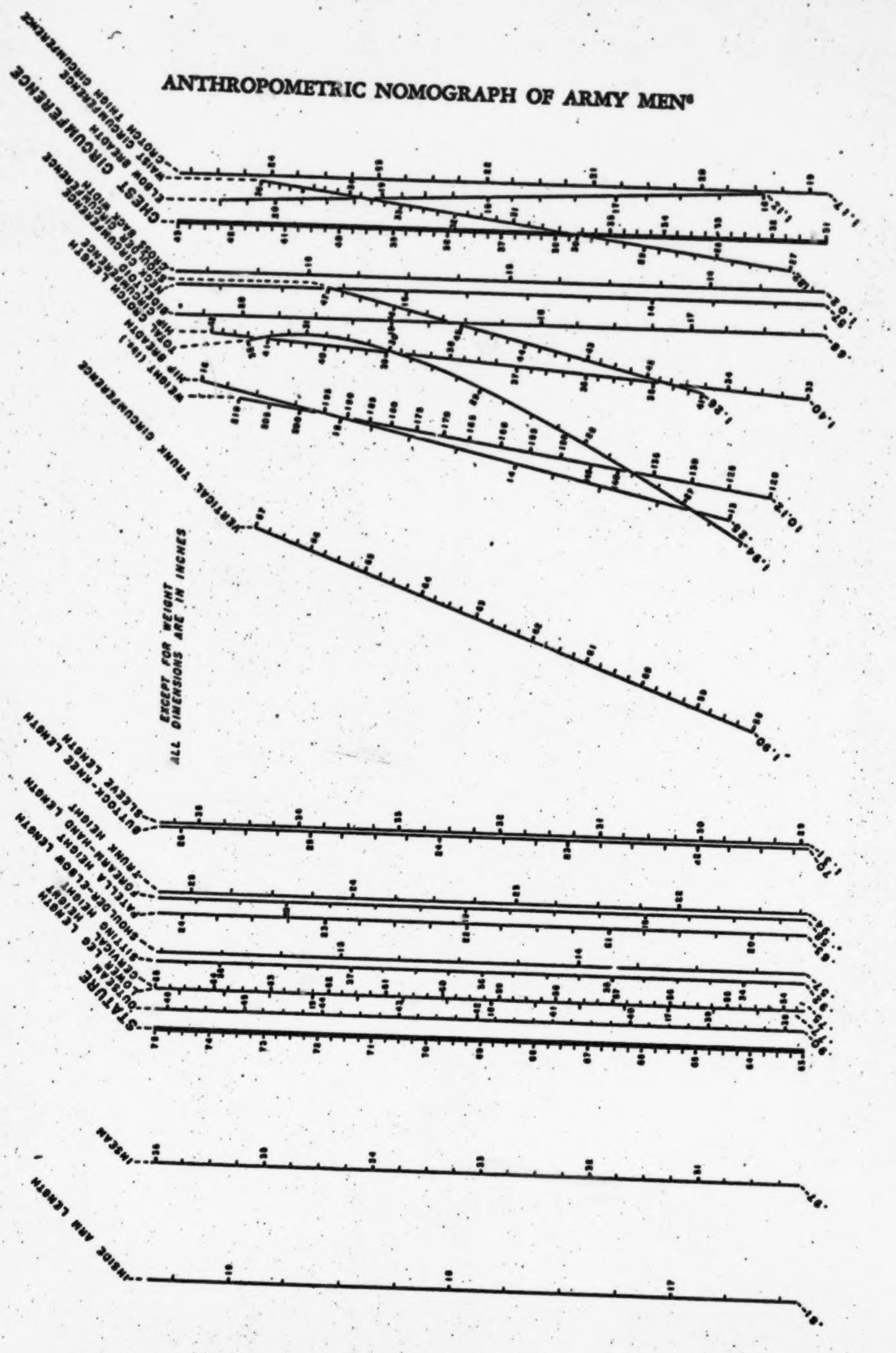


FIG. 24(b)

PRECEDING PAGE BLANK- NOT FILMED

upon an opaque reflected surface, viewing along the normal is of secondary importance. If a translucent screen is used or if the viewing eye looks directly into the optical system, normal viewing is more important and, in fact, essential if limited aperture virtual image displays are presented.

The unit should be as small as possible, commensurate with meeting screen size in order to provide maximum desk space for other use and to simplify handling and assure portability. Based on these considerations, an over-all maximum size of approximately 18 x 18 x 14" (w. d. h.) is suggested. Portability will require provisions for carrying the machine by the use of recessed handles. Protruding handles are to be avoided if possible. A machine weighing more than 20 lbs. will not be easy for a student of small stature to handle, and should be considered a maximum weight.

Because of the heavy use factor, the machine and all of its elements should be rated for continuous use, without undue acceleration of wear or parts failure.

Because of the lack of convenient power outlets, consideration should be given to possibilities for eliminating the need for power cords during classroom use. If this is not feasible, a cord length of at least six feet should be allowed. If rechargeable batteries are employed, they should be completely sealed and functionally packaged for quick and simple connection of recharging cord. Removal of battery from unit should be simple. Leakage current limits should not exceed one milliamp(14) at rated voltage.

External, as well as internal parts of the machine should be durable, free from sharp edges, attractive in appearance, yet functional. Ease in cleaning, resistance to marking with pencil or gouging with sharp instruments are also desirable. A conscious attempt should be made by the designer to render the machine as indestructible as possible within realistic cost constraints.

Ambient light levels in the school room may range as high as 15 to 20 ft. candles under conditions of normal room lighting(15, 18). Higher ambients produced by direct sunlight can be assumed under control by window blinds.

## Visual Effect Produced

An image display size of at least 9" x 9" is suggested. This will enable display of the normal page of a quarto size book. Several factors determine the utility of a projection display area, however.

If a reflection screen or translucent screen display is employed, contrast is difficult to match with that normally produced with black ink on white or buff paper. Therefore the legibility of a given type size is reduced, and in order to provide equal ease of reading text, the displayed characters should be larger; thus the screen size may have to be as large as 12" x 12" to get equivalent full page legibility. When visual image displays are used and the eye gathers light directly from the optical system, equivalent contrast is easy to achieve, and in fact, even greater contrast can be achieved, thus eliminating the need for larger image size.

Line length and line spacing (expressed in terms of the subtended angle seen from the eye point) are important, as they affect several factors in reading. It is known(16) that the eye brain combination actually reads ahead of the point seen by the fovea at any instant. The author has not found authoritative references which have tested the relationship fully, but it is obvious by simple observation of Figure 25 that line length has definite effect on reading speed. Since visual fatigue has been shown to relate to muscular movement of the eye, it is obvious that line length has an effect on visual fatigue.

How, then, should the equipment designer proceed to establish specifications for the display image size? Unfortunately, the decision is complicated by further considerations. The presentation of some tabular material and other graphics on the printed page of quarto books often

**9-PICA LINE LENGTH (38 Millimeters)**

Lorna Doone told me all about everything I wished to know, just as I questioned her; except indeed that point of points, how Master Ridd stood with her.

**13-PICA LINE LENGTH (55 Millimeters)**

Lorna Doone told me all about everything I wished to know, just as I questioned her; except indeed that point of points, how Master Ridd stood with her.

**19-PICA LINE LENGTH (80 Millimeters)**

Lorna Doone told me all about everything I wished to know, just as I questioned her; except indeed that point of points, how Master Ridd stood with her.

**29-PICA LINE LENGTH (122 Millimeters)**

Lorna Doone told me all about everything I wished to know, just as I questioned her; except indeed that point of points, how Master Ridd stood with her.

**43-PICA LINE LENGTH (180 Millimeters)**

Lorna Doone told me all about everything I wished to know, just as I questioned her; except indeed that point of points, how Master Ridd stood with her.

ILLUSTRATION OF EFFECT OF LINE LENGTH ON READING SPEED

FIG. 25

PRECEDING PAGE BLANK-NOT FILMED

requires a switch in aspect ratio (see Glossary) so that the page is longer than it is high. Optics which rotate the graphic display in the projection system are feasible, but the simple act of requiring a manipulation to rotate the image is distracting, and its use is not recommended. It is preferable that the screen be square to accommodate aspect ratios above and below unity, while making maximum use of the display area. The display image dimensions are not necessarily those of the aperture opening. For virtual image displays binocular vision requires an opening something greater than the interpupilary distance, but not necessarily the full size of the displayed image. See Figure 16.

For translucent and reflectance screens, the viewing angle is a function of the head-to-screen distance and desired freedom of head movement. Since reflections from ambient light are to be kept minimal, the image plane is commonly recessed with shade-like sides blocking off direct light from overhead lights. These further affect the viewing angle when freedom of head movement is allowed.

No single prescription can be given to selecting dimensions for the displayed image. In addition to the above considerations, a primary point in the present philosophy of programmed design is to present the material in smaller units than have heretofore been common. The effect is to require less space than normally used in conventional textbook pages. However, it would seem reasonable to include in today's teaching machine display design the possibilities that supplementary textual materials recorded in micro-image form in the same format as prepared programs for self-instruction might become available if sufficient standardization is achieved among the number of machine designs. If this happens, it is likely that the same format as now used in textbook page layout will be desirable to include in the general purpose teaching machine display.

As stated earlier in Chapter 5, an early use of color in teaching machine programs is expected. Near-true color renditions are important, as the use of color will not be limited to coding or attention-getting marks. Thus, the transmission and/or reflection properties of screens and optical elements must be chosen with the visual spectrum in mind. The light source, as an element, will have a definite effect on color of the displayed image.

If photographic materials are used for the micro-image media, the use of continuous tone images is feasible and should be exploited as these images are more pleasing to look at and are easier to achieve in production than half tones.

The resolution, the contrast, and the brightness needed to provide an image of satisfactory "quality" are interrelated. The diagram of Figure 3 shows this functional relationship, which is well accepted. Using a figure of 20 ft. candles for ambient illumination as a maximum, it is possible to arrive at the required maximum figure for projection screen image brightness.

Image contrast is defined by:

$$\text{Image Contrast} = \frac{Bg - Bi}{Bg}$$

where: Bg = brightness of the background
Bi = brightness of the image

If ambient light is reflected from the screen surface illuminated and viewed on the normal to that surface, the screen brightness in foot lamberts resulting from ambient light is:

$$Bg \text{ foot lamberts} = r\ (\%)\ I_A \text{ foot candles}$$

where r is the percent reflectance of the surface.

If a white surface of 90% reflectance is used for a screen without blinders to block ambient light off, the screen brightness due to ambient alone is:

$$Bg = (.90)\ 20 \text{ foot candles}$$
$$Bg = 18 \text{ foot lamberts}$$

If image contrast is to be .70 or higher as is necessary for "satisfactory viewing" according to references(19), then

$$\text{Image Contrast} = \frac{Bg - Bi}{Bg}$$

$$.70 = \frac{18 - Bi}{18}$$

$$Bi = 30.6 \text{ foot lamberts}$$

Note that this is considerably above the illumination level of the 4 to 6 foot candles considered(17) to provide comfortable movie screen viewing in a darkened theater.

Image brightness can be in excess of 30 foot lamberts without discomfort. Since ambient light level will vary considerably, however, provisions should be made for user adjustment of image brightness.

Notice that if a reflectance screen is used for the display, its reflectance coefficient applies identically to both ambient and projected image light so that image contrast is not a function of screen reflectance. The requirement for the projection light source to provide the necessary image brightness is thus determined by ambient light falling on the reflectance screen.

When using a translucent screen for display, the screen reflectance coefficient and ambient light level principally determine background brightness while the diffuse transmission coefficient and projection light intensity determines image brightness. For most translucent screen materials used, these coefficients are not the same and to produce a given image contrast along a source-to-eye viewing line, less projection light intensity is required than when a reflection screen is used.

If a virtual image projection system is used, the question of background brightness due to ambient light entering the system becomes a matter of the particular design,as the viewing aperture size and distance to the image plane will affect the amount of ambient light entering the system. The effects of ambient light are enormously reduced, thus reducing the amount of projection light intensity required substantially.

The designer is cautioned that a method commonly employed to change projection image brightness, namely that of changing voltage across an incandescent light source, results in considerable change in color temperature of that source, so that true color rendition in the display may be impaired. Possible use of inexpensive neutral density filters in the optical system is suggested as a means of varying image brightness.

Medium to high contrast in presentation of textual materials is desirable for clarity of reading. This relaxes the demand on high resolution, a desirable objective to minimize cost in optical elements. For a very high quality presentation, the displayed image should provide at least six to eight lines per millimeter, as viewed at a comfortable viewing distance, approximately 12 to 14 inches from the eye. A noticeable deteriorated image will occur if resolution is less than this amount. However, legibility in most cases of most type styles can be expected, with resolutions as low as 2.5 to 3 lines per millimeter. Resolution in excess of 6 to 8 lines per millimeter may result in more expensive components than necessary, for the human eye cannot detect spacial frequency components in excess of that amount.

Image Projection System

A basic element in the projection system important to choose properly is the light source.

If either the suggestion that 35mm roll film or the suggestion that battery operation (or no power at all) be followed, the designer will conclude that the projection sub-system is one of projecting a transparency rather than an opaque object. Although it has been demonstrated(20) that a workable teaching machine can be built to render a visible display without incorporating an internal light source of any kind, this approach has several limitations. The light gathering aperture providing illumination from the rear for the transparency must be considerably larger than the area of the displayed image, even if a deep light shield is used to surround the projection of the displayed image, as seen by the eye, with head in normal position. If the image displayed for viewing were 9 x 9 inches in size, the light gathering aperture would become excessive and cannot be provided within the overall size requirement. Further, the light gathering approach is highly directional. As everyone who has used the inexpensive 2" x 2" slide viewer knows, if a unit utilizing light gathering design is so placed as to bring in direct rays from a light fixture, the image brightness will be much greater than when the aperture looks at a dark wall surface. These are definite limitations which render this approach unacceptable for the general purpose machine for use in the secondary schools, in the opinion of this writer.

In choosing a conventional transparency projector light source, the desire is to obtain as concentrated a point source as possible. As described in Chapter III, the light collection system in the projector housing collects as much light as possible from the focal point of the optical elements. If the light source is distributed over an area not in the focal plane, much of the light will be lost as stray light. In the case of incandescent bulbs, where hot glowing wire emits the source light, a small tight coil filament source can be obtained such as those shown in Figure 26a. Because of the limited size for heat dissipation, these are not available in high wattage. Where a greater power is required for more intense light, a design commonly used in projectors of the 300 watt, 500 watt, and above employ a bulb construction as indicated in Figure 26b. Although their use is less efficient, as a greater percentage of the emitted light is lost, additional light can be channeled into the projection system.

With any light source employed in the projection system, the source must be properly positioned. This is important, as bulbs must be replaced upon failure and are usually positioned by a base socket connection. Consequently, bulbs manufactured for projector use are carefully controlled to locate the source element accurately with respect to the bulb base. Since the precise position of the glowing elements change with mounting position

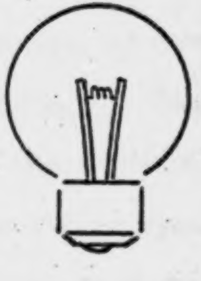


FIG. 26(a)

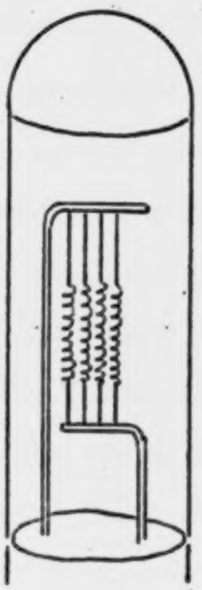


FIG. 26(b)

TYPICAL PROJECTOR BULB CONSTRUCTIONS

of the bulb, because of sagging under intense heat, proper operating positions specified by the bulb manufacturer must be observed.

Bulb life is greatly foreshortened by over voltage and life is increased greatly if operated below rated voltage. Use of lower voltage, however, reduces the color temperature of the source. This must be taken into account if true color renditions in the projection displays are designed. For this reason, it is suggested that variable voltage control not be used as a means of adjusting image brightness. On the other hand, if true color rendition is of no concern, this is a simple and convenient way of adjusting image brightness. If a fixed voltage is used below rating, the proper blue tint filter will provide the necessary color correction for true color renditions.

Excessive image brightness is a waste of power, a matter of considerable importance if a battery-operated unit is contemplated. In projection systems using light sources above 50 watts, heat dissipation must be given consideration in the design of the projection unit. This is important in order not to exceed the maximum envelope temperature of the bulb as recommended by the manufacturer. In addition to proper dissipation of heat around the light source, it may be necessary to employ a heat filter in the condensing system (see Chapter III) to filter out the longer wave length heat rays, which would otherwise be focused at or near the plane of the transparency. Even though temperatures at the surface of the transparency may be far less than that producing permanent damage to the record material, a change in temperature of the transparency can cause great difficulties in maintaining the transparency in a flat object plane. The tendency is for the transparency to buckle under heat and either distort the image over the field or to pop into a different plane, thereby defocusing the image each time a newly positioned frame heats up. This condition cannot be tolerated as the user cannot be asked to refocus from frame to frame. The use of glass flats to sandwich the film is not recommended as a cure for this difficulty, as it greatly increased the complexity of the film advance mechanisms, since the film cannot be simultaneously sandwiched and moved from frame to frame without producing severe scratches.

The requirement for high quality optics in the projector housing is not great. Several flaws and imperfections can be present without noticing any effect in the projected image. These defects commonly cause a slight reduction in light intensity, but for purposes of visual display, this is not a critical factor.

Many good projection lenses are available from a number of manufacturers and are relatively inexpensive, as compared to camera lenses. In order to obtain satisfactory screen illumination, projection lenses must have relatively large apertures. Very fast projection lenses are available even at low cost, since the angle of view of projection lenses is smaller, and optical problems arising from angular coverage are less complicated than in the case of the camera lens. Projection lenses require flatness of field and good visual color correction to assure efficient and true color rendition. Barrel mountings of the optics should be anti-reflection finished in order to minimize the reflection of stray light which deteriorates image contrast. Coated lens surfaces use a thin hard coating of magnesium flouride to reduce surface reflections, thereby reducing flarelight and spots. The image quality projected by lenses of this type is improved with higher contrast and more detail in the shadows. In particular, the shadows in color pictures have greater color purity. Where continuous tone is of no interest and simple line text is projected, these particular image quality features are not so significant. However, wherever the need for high contrast is present, the coated lens systems should still be used. When all the elements, including condensors of a projection system are coated, image brightness is increased as much as 50%. Coated lenses can be identified by the slight bluish tint seen by reflected light.

The lens mounts employed in precision cameras are costly and contain parts machined to high tolerances. These are not at all important in a projection display lens and even plastic or light metal mounts are perfectly adequate.

Of the three types of graphic display systems described in Chapter III, the reflection surface display appears best suited for the teaching machine application for the following reasons:

☆ Freedom in head movement is possible in contrast to the more restricted fixed head position requirements of visual image systems.

☆ True color rendition is easily achieved on screen materials having a white mat surface.

☆ Glare common in translucent screens along the path from eye to projection lens is not present.

☆ The problems of ambient light impinging on the same display surface can be controlled to maintain adequate image contrast without excessively high image brightness.

If the reflectance screen surface is used, choice of material must provide for easy cleaning, as heavy use will surely result in screen surfaces becoming dirty and worn from pencil marks, etc. when accessible to the user. A good design would minimize direct access to the reflectance screen.

If the material selected is tested for reflectance of the visual spectrum with a spectrophotometer, failure to give true color rendition can be quickly detected. For comparison purposes, the spectral reflectance of manganese oxide, often used as a reference material for nearly perfect white light reflectance, is shown in Figure 27.

If a translucent screen design is chosen, the designer is encouraged to investigate the suppliers of screen materials thoroughly as developments are continually underway to develop a better translucent screen material. The object is to choose a white translucent screen which will disperse light efficiently, yet uniformly, over the viewing angle, while at the same time rendering a true color image.

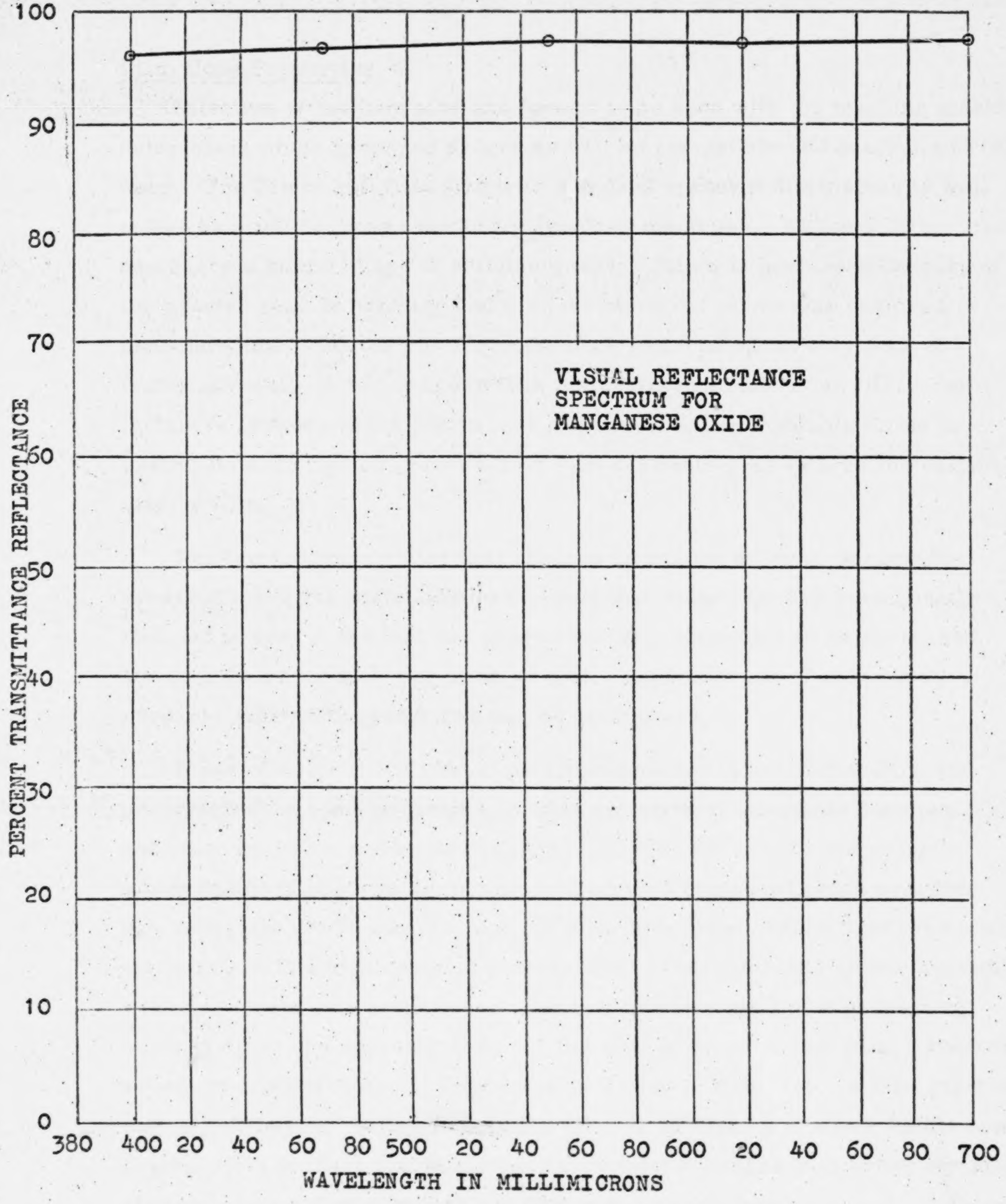

100
90
80
70
60
50
40
30
20
10
0
PERCENT TRANSMITTANCE REFLECTANCE
VISUAL REFLECTANCE
SPECTRUM FOR
MANGANESE OXIDE
380
400
20
40
60
80
500
20
40
60
80
600
20
40
60
80
700
WAVELENGTH IN MILLIMICRONS

FIG. 27

PRECEDING PAGE BLANK-NOT FILMED

## Film Copy Properties

Selection of the film size and format to be used with the teaching machine determines which prepared programs will be compatible and available to the user. The 35mm roll film size with standard sprocket dimensions is well suited as a micro-image media for teaching machines. Although 16mm film can carry a micro image of suitable quality, 35mm is preferred because of the greater ease in handling and the less stringent tolerances required in associated mechanisms reading code block patterns which may control frame advance. A full range of film materials available from film manufacturers is made in the 35mm size and services are available to the program publisher for the production of film duplicates, as well as the original master film.

For the program writer that chooses to employ color in the graphic presentation of his material, he will find that color film has been greatly reduced in cost in the last few years to a point where it can be purchased and processed for $20 per hundred feet. A greater use of color in film prepared instruction programs can be anticipated.

If use of color is not chosen as a requirement in the preparation and presentation of some programs, a wide selection of materials becomes available for black and white displays. Besides the more conventional silver halide film products, lower cost organic compound photo sensitive dye materials are in common use for micro records. Diazo film previously made only with a high contrast photosensitive characteristic is now available with continuous tone properties. The raw film material plus processing costs of diazo are approximately 1/2 the cost of silver halide film. Another recent microfilm material introduced is Kalvar. This, too, is less expensive than silver halide, falling in between the cost of diazo and silver halide products. Both of these materials can carry micro-images with better wearing characteristics than silver halide film without a surface hardener. Although efforts are underway to develop a hardener for silver film to accomplish equivalent wearing characteristics, no evidence has yet been presented to demonstrate equivalent or superior wear characteristics from silver film.

Kalvar material will retain an image of great permanence (hundreds of years), as well if not better than silver film. However, it is a characteristic of the diazo dye materials to fade with time. Fading is accelerated by exposure to ultraviolet, and if a processed diazo film record is left in direct sunlight for two to three months, the image may fade beyond use. However, the material can be handled in normally lighted rooms for years without fading. If the program film is kept under good control, both in storage and in use with the teaching machine, a diazo film will provide more life in the permanence of the image than required. The two characteristics of diazo which are espeically attractive for this application are, (a) its wearing characteristics resulting from the fact that the photosensitive material is imbibed directly into the film base material, and (b) it is a non-reversing process so that if a camera negative is used as a master for generation of duplicates, the duplicates resulting from direct contact prints from the master will also be negative. Thus, if the suggestion that negatives be used for viewing full text there is less need for an intervening step of producing a positive from which negatives could then be made. A reversing diazo film now under development will enable the making of positive duplicates where it is desired to present photographs or other material which is preferably presented in positive tone form.

Although the Kalvar product accomplishes long wear characteristics, permanence of micro image, and lower cost than silver products, it is best applied only when adequate consideration has been given to its properties in conjunction with the particular projection system being designed. In short, the image contrast in a graphic display of a micro object on Kalvar is a function of the relative aperture of the projection system projecting this image. This is not normally the case in projecting micro images, but in the case of the Kalvar material which accomplishes the projection of light and dark areas through different degrees of light dispersion, relative aperture of the projection system is significant. With most photographic film dark areas in the display are accomplished by the transparency absorbing light rays, thereby preventing their projection onto the display surface. The Kalvar film photographic properties record images in a different way. The corresponding dark area in the micro image of the transparency does not absorb the source light ray, but diverts its direction so that it does not get directly to the display screen surface. However, the dispersed light passing through the Kalvar micro image is not entirely captured and some of this gets to the display area contributing to an increase in background illumination. When relative apertures are small in the projection optics, the amount of light reaching the display is decreased, thus increasing the image contrast of the display image. If relative apertures below f/4.0 are used, projection background densities as high as 2.0 can be achieved and adequate image contrast maintained. Projection lenses with smaller apertures can give even higher contrast, but they are not recommended as this increases the projection light source power required.

Film will become disassociated with the storage device (box or can) which not only protects the film during storage, but identifies with a label. Program films should have adequate markings at both ends of the roll to identify the subject matter, title, publisher, etc., comparable to the catalog information given to books when accessioned by the library. The easiest way to accomplish this is photographically in the original or film master. However, acetate inks are available that can easily be applied to any acetate film base with acetagraph pens and other instruments. Character size should be large enough for reading by the unaided eye.

Frame size and format selected for the presentation of a program film will also affect the compatability of that program with a variety of machines. After studying the requirements for graphic displays in a general purpose machine for the secondary schools, the writer concludes that the 35mm single frame size as defined by ASA standard PH 3. 39-1961 (Appendix E) is more than adequate. If the dimensions of the projected image are chosen as 9 x 9 inches, the magnification required is between 9x and 12x depending on how much, if any, of the usable image area between sprocket holes is used for other purposes. Branching programs may require a code pattern in addition to the display information for control purposes. Although this information could conceivably be on another medium such as punch paper tape and properly synchronized, , this would be troublesome to the user and expensive. Thus, where code patterns for frame control are needed, they are best recorded directly on the film along with the graphic image. The location of the frames with respect to the sprocket holes is defined by the ASA standards for 35mm movie film.

The matter of code pattern and placement has little or nothing to do with the graphic display directly. These design choices depend on the detector components selected and the functions performed for controlling the film advance. Even here, however, the day of standardization will be looked forward to for, as elsewhere, the question of compatibility is met head on.

### Insertion of Program Film

Insertion of the program film should be simple.

No matter how simple the task is, a diagram and instructions should be affixed to the equipment in an obvious location.

The method of engaging the program film with the transport mechanism is important as this is likely to be the point producing greatest film wear. Removal of one program and replacement by another should be a rapid and simple task as the same machine may be used by several students with different programs for instruction during one. day. The change of program materials is not an operation which necessarily needs to be done by the student while in operating position. Movement of the film reel during operation of the frame to frame advanced mechanism as the program is presented, is preferably kept out of sight of the student. A moving object tends to command attention.

Cartridge loading of roll film is to be desired. Use of cartridges minimizes the direct contact of the program film with the hands thus reducing fingerprints and dirt accumulation through heavy use. It can also simplify the problem of loading the machine thereby minimizing wear and tear resulting from those who fail to learn how to properly thread the film. Cartridges further protect the program film in storage both on the shelf and at all times prior to manipulation by the frame advance mechanism in the machine. In comparison, film not in cartridges will be dropped, inspected, removed from reels, rewound backwards and otherwise be subjected to misuse to a far greater degree than if the film is kept in the cartridge except when in the machine. Although the use of cartridges undoubtedly will add some cost to the selling price of programs, the savings resulting from greater protection in handling of the program films suggests that the wise use of cartridges can actually be an economy for the user through extending the life of the program film copy.

### Frame Locator Mechanism

Included in the locator mechanism are the film transport mechanism, the means for finding and stopping at the frame of interest and the means for properly registering the frame once it is found. In the Mentor machine built by Recordak the student manually advances the strip film program from frame to frame. The operation is so simple that it compares with the simplicity of button pushing. This method of frame selection works equally well for linear or branching programs although the ability to stop at the wrong frame through error or curiosity is under the control of the student rather than the machine. If the manual task for frame advance can be achieved in a roll film transport system with no greater requirement of the machine operator than perhaps that of the Mentor using a flat sheet format of four strips each, a manual film advance would be acceptable. The advantage of a manual film advance, of course, is the probable elimination of a motor drive in the unit which would considerably lower the cost and the requirement for power.

Functionally, the requirement for film advance alone does not suggest a basic requirement for motorized drive. Advancing from frame to frame when exclusively under the control of student pacing requires some action on his part to initiate advancement. The energy required to advance 35mm roll film from one frame to another within a 12 to 18 inch strip length is hardly great enough to place a significant demand on the machine operator. If this power requirement can be obtained from the response mechanism as it is actuated, and the necessary control action is initiated to identify the next proper frame for the machines selection mechanism, it would seem possible to design a machine powered by the operator except for the light.

However, the possibilities for machine pacing in the presentation of program material have been investigated and shown to have some merit(21). Also, if the possibilities for incorporating audio presentation of material simultaneously with that presented on film, the form of synchronization may require motorized drive. These points suggest the advisability of incorporating motorized drive in a teaching machine designed for general purpose in secondary schools. Except for the projection lens, the drive motor will be the single most costly element employed in the machine. Its choice, if a battery powered unit is needed, should be made on the basis of life, power drain, cost, and physical size, in that order. Starting voltage of the motor will also be an important consideration as failure of the motor to start will be the point at which a given battery charge will for practical purposes be depleted. In most electronic devices that are battery powered, separate batteries are employed for the motor circuit. Since speed over a wide range for transport operation is not critical, the motor battery can be drained to a point producing considerable voltage drop. Other electronic circuits employing transistors and photo sensors as may be required for control circuits in the frame advance mechanisms will require less power, but more fixed voltage, thus necessitating a separate battery. Nickel cadmium battery units presently employed in portable electronic equipment are usually packaged in a single container with a multi-prong plug thus for user purposes, becoming a single unit.

If machine controlled pacing is employed, a timing mechanism is obviously necessary in the equipment. This will also require power and should not be overlooked in estimating the total power required to operate the unit.

In a linear program sequential frames are arranged adjacently. As the frame is advanced it seems unnecessary to provide screen blanking. However, in branching programs where several intervening non-pertinent frames must be skipped, the use of a blanking mechanism during film advance is suggested, unless the film is advanced at such a speed as to make it impossible to glimpse the content of intervening frames. Turning the source light off as a blanking means is not recommended, as the life of most light sources is considerably reduced by frequently turning the circuit on and off. The life of incandescent filaments frequently operated in on-off applications can be lengthened if operated at reduced voltage. The effect on color temperature at reduced voltage must be taken into account, however, if true color rendition is to be achieved in the display.

Stolurow(5) has drawn attention to the fact that the time interval between the choice of an answer and the associated feedback will vary not only from learner to learner, but also from problem to problem. He observes:

> "From results of laboratory studies of delay and feedback (either as reinforcement or as knowledge of results) it is clear that feedback should be immediate for maximal efficiency learning to take place. Thus the location of each answer page should be as to minimize the amount of page turning. The degree of scrambling is an important determinant of the feedback interval and should be considered with respect to the need for efficient learning."

In addition, then, to the obvious need to eliminate the student's observance of other material, the equipment designer must assure rapid selection of the next frame. This, too, strongly suggests the advisability of incorporating motorized film advance mechanisms design.

Use of the standard 35mm sprocketed film simplifies the design problem for frame-to-frame advance, as well as registration of those frames. This benefit results from a specified relationship between the sprocket holes and frame location. This same specification applies to several types of cameras that may be used in filming the original master and in the use of duplicating equipment to assure proper alignment of all duplicate films. Thus, the whole system from camera to teaching machine is easily controlled when standard sizes are used. If sprocketed film were not used, some form of mark would be necessary to signal an automatic positioning device to assure proper locations. If manual film advance drive were used, an arrangement of detents might be designed to assure proper frame registration. If flat sheet or film strips without perforations are used, some form of carrier with fixed assigned frame positions, identified by mechanical detents can accomplish frame registration. These latter methods, however, are considered less desirable and more complex than that achievable through the use of standard sprocketed 35mm roll film.

Operator Controls

The operator controls, as distinct from the response functions, are preferably located out of sight on the side (or top) of the machine or under a hinged cover on the front panel. The required controls include an on-off power switch, a power-on light, a focus control for the projection optics, an image brightness control, and, if used, a pacing control. Elimination of all activity from the front panel other than those controls and displays contributing to the learning process is a basic objective which seems to have been overlooked or ignored in the design of many teaching machines on the market today.

The internal operations and mechanisms of the device should not be accessible to the student. If necessary, a locked access door should be provided, although screw fastened panels are equally suitable and at lower cost. Since program reels will be changed frequently, it should not be necessary to open access doors to change programs.

### Student Response Provisions

Little can be said by this writer on the best approaches to incorporating response means. It would seem that the optimum choice for response means depends greatly on the nature of subject material and precisely what the student is intended to learn, as well as the associations which the instructor may wish to create within the act of responding. Spelling, for instance, would seem preferred by some to be taught by a teaching machine which required the overt response of the student to actually write out the answer. A frequent association between writing and spelling seems indicated by the fact that many times, a person will write out a word before commiting himself as to the proper spelling. When asked why he does this, the answer is, "I want to see what it looks like in my own handwriting."

Task-related effort has been shown(13) under certain circumstances to have a facilitating effect on perceptual learning. A drawing effort related to the test patterns presented showed maximum facilitation suggesting that task-related overt written responses might be more effective in teaching some program materials than multiple choice button pushing..

The multiple answer array of response buttons has been used with positive results in the presentation of considerable auto instruction program material. It appears to be a satisfactory general form of response, and therefore might best be built directly into the front panel of the general-purpose teaching machine.

If a modular approach is used in the design, it seems feasible that alternate response units can be built to work in unison with and be connected to the basic display unit with which this report is primarily concerned. Once more, the design of special purpose features of all kinds for inclusion into the basic unit would result in unwarranted costs. The building-block approach, however, will offer special purpose features when desired without throwing away or duplicating the basic general purpose features. This suggests that the basic display unit should include a multi-prong connector to provide power and exchange signals with external auxiliary devices.

## Other Factors to be Considered

There are, of course, a number of other factors in the design of teaching machines beyond those covered in this chapter. We have said nothing about the means for recording the student's incorrect responses. Neither have we discussed the comparator which compares student's response with the correct answer in order to determine the next frame for presentation. These devices or circuits, although basic, have little direct connection with the graphic display. For that reason, they have not been discussed in this report.

It is true that much remains to be learned about the basic mechanism of the learning process. When more is known, a better basis for machine designs will be available. This point, however, cannot be used to support the present trend in machine design and development. Most of the present machines and others currently being introduced are much alike in function. They differ widely within a particular class of sophistication, however, in details of their design without significant value differences to the user with the result that today there is nearly complete lack of compatibility between programs and machines, except where both have been produced by the same supplier. The field is in need of discipline to establish standards that will allow making best use of what is known about the elements of good machine design.

## CHAPTER VII

## DETAILED CONSIDERATIONS IN THE PREPARATION OF PROGRAM FILMS

### Original Copy Preparation

A great amount of work and planning effort must go into the preparation of a program for auto instruction. Stolurow (5.) describes the programming processes and the concepts and techniques of program preparation. The array of concepts and techniques is substantial and should not be attempted by the amateur, but left to the professional steeped in the methodology of education along with familiarity with the teaching machine form of presentation.

These considerations dwell on the content of the program, its stimulus and response elements, the size of these elements, and their sequence of presentation. As the program is being written, certain decisions relating to graphic displays will be implied. The program writer is best equipped to decide where the use of graphics of one kind or another will assist in presentation. He should therefore be familiar with the methods of reproduction and graphic arts possibilities to the extent that will enable him to decide on the practical use of supporting graphic materials. Otherwise, the program may be a monotonous and less effective presentation of text alone. One of the objectives of this chapter is to present in capsule form, a description of the variety of graphic arts techniques that can be applied economically where the form for presentation of the prepared program makes use of photographic film.

Once the program has been written and the over-all program sequence and sub-sequences have been determined, the "copy" is ready for review by those concerned with its typographic presentation and the addition of graphic arts work for effective communication in graphic displays. The over-all problem of preparation of the master for duplication can be subdivided into problems of:

- ☆ Frame lay-outs
- ☆ Type style and size selection
- ☆ Typographic composition
- ☆ Planning and preparation of art work

In arriving at a satisfactory page or frame lay-out, the program writer will have a good deal to say about the final selection. This is much more true in the case of auto instruction programs for teaching machine use than is true for the preparation of textbooks. One of the basic elements of teaching machine presentation philosophy hinges on the size of the step presented, and therefore, the content of a single frame in the program. If a frame is to contain both text and supporting graphics such as a photograph or diagram, the program writer should specify the relative relationship of the text and supporting graphics. The reading habits of western civilization establish the sequence of items presented in the frame scanned by the student. The arrangement of frame content is therefore as important to the program author as is the arrangement of frame sequence.

In the selection of type style and size, a publisher of auto instruction programs will turn to experts other than the author to determine the optimum selection. Considerable experience has been accrued by publishers on the relative effectiveness of particular type styles and size of type best applied to presentations for readers of different age groups. Information is also available on the relative effectiveness of styles and sizes for adults. A quick review of the subject will show that many of the programs prepared by the smaller firms have been assembled with limited facilities making use of typewriters or mechanical lettering equipment for the preparation of text materials.

Considerable development of low cost type setting equipment providing a wide selection of type styles and sizes has been underway in recent years. The interested reader is encouraged to investigate these devices as low-cost approaches to providing more effective composition.

Once the decisions of type style and size have been made, as well as the decisions of over-all page lay-out, an experienced type-setter can apply his skills in arranging words and symbols in the running text according to effective practices in choice of line length, inter-word spacing, indentation, line-to-line spacing, justification, hyphenation, etc.

Where supporting art work is specified, considerably more freedom of choice exists than typically true for the preparation of printed text. Continuous tone photographs can be used freely since the entire reproduction process (through, to, and including the distributed copy) is photographic, therefore requiring no conversion of continuous tones to half tone plates. Where color is employed, the original and duplicate films will have to be color film and therefore no special graphic art processes are required to produce separation negatives normally needed to produce full color reproduction by conventional printing techniques.

The common practice of making original art work larger than normal size may still be desirable where the highest quality and precision in art work presentations is desired. Where continuous tone graphics are used in a photo instruction program, the camera film employed must have continuous tone properties and thereby will be sensitive to all smudges and erasures. If no continuous tones are used in the graphics employed, a high-contrast film for either the camera original or a subsequent duplicating high-contrast film may be chosen to wash out the effect of smudges and corrections in the original. This consideration is in distinct contrast to common graphic arts techniques employed in the preparation of printing plates where high contrast photographic materials are purposely employed throughout to reduce the requirement for error-free original copy for all line text graphics. The use of a single photographic emulsion in the camera film roll for the teaching machine program master removes the possibility of giving this treatment to line text if the film is also to carry continuous tone photographs.

Camera Operations

One of the advantages in using standard 35mm roll film is the availability of camera equipment that assures perfect control between frame location and the sprocket holes in the film. Special purpose equipment can, of course, be developed. Even movie cameras can be modified to meet this requirement. Cameras known as "animation cameras," produced for the movie industry to produce titles, include all of the features necessary to film program frames in proper sequence to assure both proper frame location with respect to sprocket holes and means for proper registration of graphics within a given frame.

A 35mm animation camera, built to the highest quality standards, will cost about $35,000 new. For those producers of teaching machine programs who do not contemplate purchase of such a facility, rental is available on the same equipment in most large cities where television studios make use of this same equipment in preparing TV film titles. A representative rental rate available in the Boston area is $15 per hour(higher in New York).

Registration of the information presented in each frame is controlled by punched holes in paper to be used for the preparation of original copy. An alternative technique is to prepare original copy on regular paper which is then attached to the registration sheet. If a separate page is thus prepared for each frame of the program prior to camera operation and the pages are arranged in proper sequence, the time required for the camera operator to completely film a program of 1000 frames is approximately eight (8) hours, providing no color is present. If color is used in the graphics, equivalent speed of filming can be done only if original graphics have been made to assure that proper color balance is achieved with the same, or similar, lighting conditions at the camera. If color graphics have been accumulated from a variety of sources and not examined closely for exposure requirements, the camera operation will be slowed down accordingly.

In addition to the information content of the program in each frame, it is likely that a code pattern will be set up necessary for frame advance control in the mechanism of the teaching machine with which the program is to be used. This again is best attached as an individual pattern for each frame photographed so that the entire program can be checked without tieing up the expensive camera equipment. If the camera is owned and not being loaded with production, however, a simple means for introducing the code pattern at the copyboard may be developed with consequent cost savings. This can be done by hinging small segment panels which flip into the frame code area, as long as proper instructions are written at the bottom of each original copy page for setting up the code.

Although camera operation is simple, its proper adjustment to assure optimum exposure, correct focus, precise registration and correct magnification are best performed by someone experienced in applied photography and knowledgeable about the specific equipment employed. Since proper exposure depends on so many variables, it is likely that each new combination of equipment settings, original copy and selection of camera, as well as duplicate film materials, will result in the need to produce trial exposures under controlled conditions to determine the optimum setting for the over-all photoreproduction system.

### Film Processing

Part of the over-all photoreproduction system is the film processing step. The processing of camera negative film by existing service organizations is an inexpensive consideration. Furthermore, standards have been adopted as the result of modern uses of microfilm under Department of Defense specifications which, if employed in the service organization, will assure consistency and control of image density of the film processed. If substantial activity exists in the production of either camera negatives or duplicate films, the reader may consider the purchase of his own facilities for film processing. This, of course, can be done, but is advisable only if quality control standards are maintained to remove any substantial variation in film processing. An unprocessed exposed negative roll of film, containing 1000 images for instance, represents a substantial labor investment, leaving no room for trial and error in achieving satisfactory film processing.

If color film is used, it is recommended that a service organization be employed to process the color film, as adequate facilities for control of proper processing conditions require a considerable investment in both capital and experienced personnel. It is advisable to consult with color service organizations about the over-all program of filming in color and subsequent duplication and processing of duplicate films for distribution.

## Error Edit and Splicing

The master film from the camera should undergo careful inspection before it is used to make duplicates. Film splicers are available that will produce strong heat splices(22) without loss of a sprocket hole or overlapping adjacent frames at the splice point. Thus it is possible to incorporate corrections on a frame-to-frame basis without refilming the entire program roll which would be an expensive proposition. Because of human error, it is not feasible to correct camera operator errors by complete refilming of the roll. The only practical approach is to employ the splicer in the master roll.

## Duplication and Distribution

One of the important advantages in publishing in roll film form, as compared to the conventional printed page, is the ease with which masters can be used to produce additional copies on demand. This has the important consequence that there is no need to produce large numbers of copies in anticipation of heavy demands in order to gain the cost reduction of large volume production. The obvious consequence is the elimination of inventory, investment and storage space. In short, duplicate copies can be made in small quantities and reproduced as the demand comes in without paying a premium or higher price for the cost to produce subsequent duplicate films.

Quality control procedures apply equally well to the production of film duplicates as described above for camera masters. The investment of equipment and experienced know-how is relatively small, being in the order of $1000 or less for equipment. To adequately determine a satisfactory product, even this investment may be eliminated if arrangements are made with outside service organizations to include in their responsibility the furnishing of readings taken from their own quality-control equipment and made available to the purchaser to assure the product meeting his specifications.

# CHAPTER VIII

## SUMMARY

The highly active development efforts underway in the field of teaching machines seem to be in need of

- ☆ An authoritative (informed) statement of requirements
- ☆ A compilation of performance characteristics about existing equipment
- ☆ A set of standards to channel the development efforts into useful directions

This report presents an analysis and discussion of the physical requirements for teaching machines that present graphics via projection displays. This is felt to be an area of major importance because of the combined need to miniaturize program packages and because of the increased flexibility offered by these systems. Despite the rather controversial nature of the early concepts in the development of teaching machines, it may nevertheless be concluded that a solid and well founded engineering basis for defining display requirements is at hand. Although the display unit is but one of the significant elements of a teaching machine design, it is the dominant element insofar as cost and determination of compatibility among machines and programs from several sources is concerned. Besides the presentation of requirements for the display elements of the design, several other characteristics were set forth in this report as requirements for teaching machines to be used in secondary schools. These included questions of size, portability, weight, power requirements, durability, and ruggedness of design, facility of use, location and arrangement of operator controls, and others.

Another element of the Teaching Machine subject discussed as directly related to the analysis of display requirements is the production of program film. Since a machine without a program is of no use, the over-all cost of an auto-instruction system very much depends upon the cost of program film production. It is concluded that photographic film as a medium for program publication offers advantages over conventional printing techniques resulting from

- ☆ Lower investment in production equipment
- ☆ Speed and ease of producing duplicate copies on demand, eliminating need for large inventories and long press runs
- ☆ Increased flexibility in presentation technique, including and in excess of the full range of techniques now employed in graphic arts.

As the result of examining the considerations determining display requirements as well as the production of programmed films, this report concludes that the standard 35mm roll film (sprocketed) is ideally suited for use in teaching machines. There appears to be no sacrifice of user requirements resulting from this selection and after considerations in this study, it appears to offer several distinct advantages of

- ☆ Ease of handling
- ☆ Excellent image quality at moderate-to-low cost
- ☆ Wide selection of photographic characteristics available, resulting in
    - · Existing equipment for production and processing of programs widely available and under control of well accepted standards
    - · Widely available services to perform several specialized functions in the production of program films.

Other elements in the teaching machine, having an impact on physical design, are recognized as requiring further freedom for development as more insight is gained as to their relative importance. The physical design requirements of the response panel, the confirming mechanism, and the reinforcement means are not safely frozen without further research and determination of the relative effectiveness of alternate designs. These are not engineering considerations, primarily, but have a predominant role in the man-machine relationship of the teaching machine design, and therefore await a better understanding of the learning process. A building block approach is suggested as feasible, however, which will permit the development of alternate requirements in these three areas without further deterring the adoption of display requirements and standards.

It is concluded from observation of the characteristics of machines available at present that a great deal of lost effort is bound to result because of the failure of the industry to achieve compatibility among the programs and machines. It is concluded from this study that a technical basis exists for getting on with the development of standards in order that compatibility can be encouraged and ultimately assured to a point where a general purpose machine concept can serve the largest portion of the secondary school teaching machine need. The general purpose machine may then be supplemented by special purpose machines designed to present special materials where needed. A cursory examination of the designs of existing machines indicates that some design decisions have been made to prevent compatability, hoping to achieve uniqueness in a good product, thereby capturing more of the market.

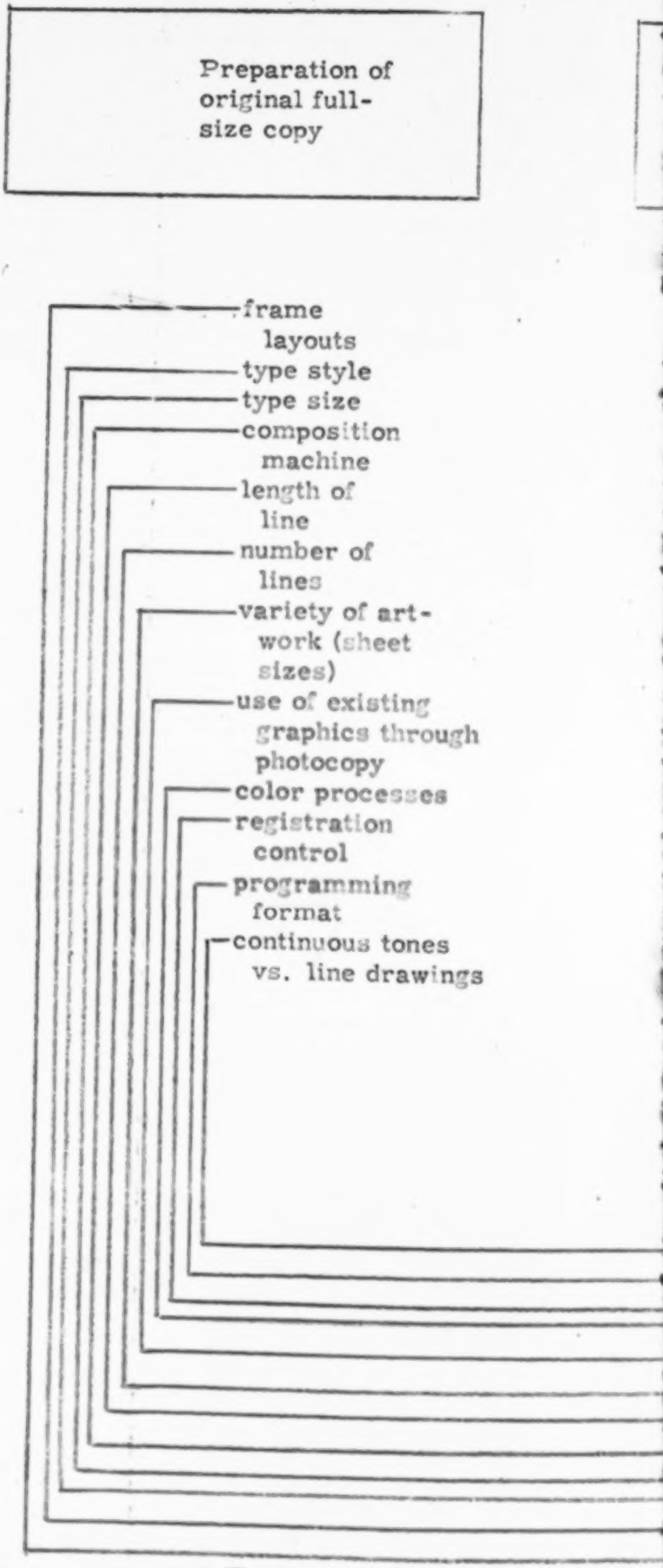
In the industrial field, standardization has naturally evolved under a system of self-government in many industries. It is but one example of man's evolutionary conclusion that he must impose limits on freedom of choice even if living in a free society or there will be no society.

Before standards can be developed and adopted in a teaching machine field, facts on existing machines must be collected, even in those areas of design considerations which basic requirements should be clear enough now to achieve general adoption. Once the requirements have been adequately stated and shown to be correct and without bias, the next step is to obtain factual information on the existing equipments through objective analysis in order to make the most of what has been done. Once these facts are assembled, the best course of action in adopting and developing standards that can be followed in the field is possible.

The development of standards, their adoption and use in new designs is a time-consuming process. It is doubtful that the demand for teaching machines in the secondary schools will wait very long for an orderly progression of these steps. In the meantime, decision makers, anxious to get under way in applying program courses of study in their curricula, need to be better informed on the properties of machines whose uses they may be contemplating. Information supplied by even the most reputable manufacturer is a doubtful basis for an evaluation of alternate choices. The joint statement prepared by the committees of the American Psychological Association, the American Educational Research Association, and the Department of Audio-Visual Instruction of the National Education Association appears in Appendix F. Quoting from this statement,

> "The use of self-instructional programmed learning materials in teaching machines and similar devices represents a potential contribution of great importance to American education, but this contribution can be realized best only if users have information with which to evaluate self-instructional materials."

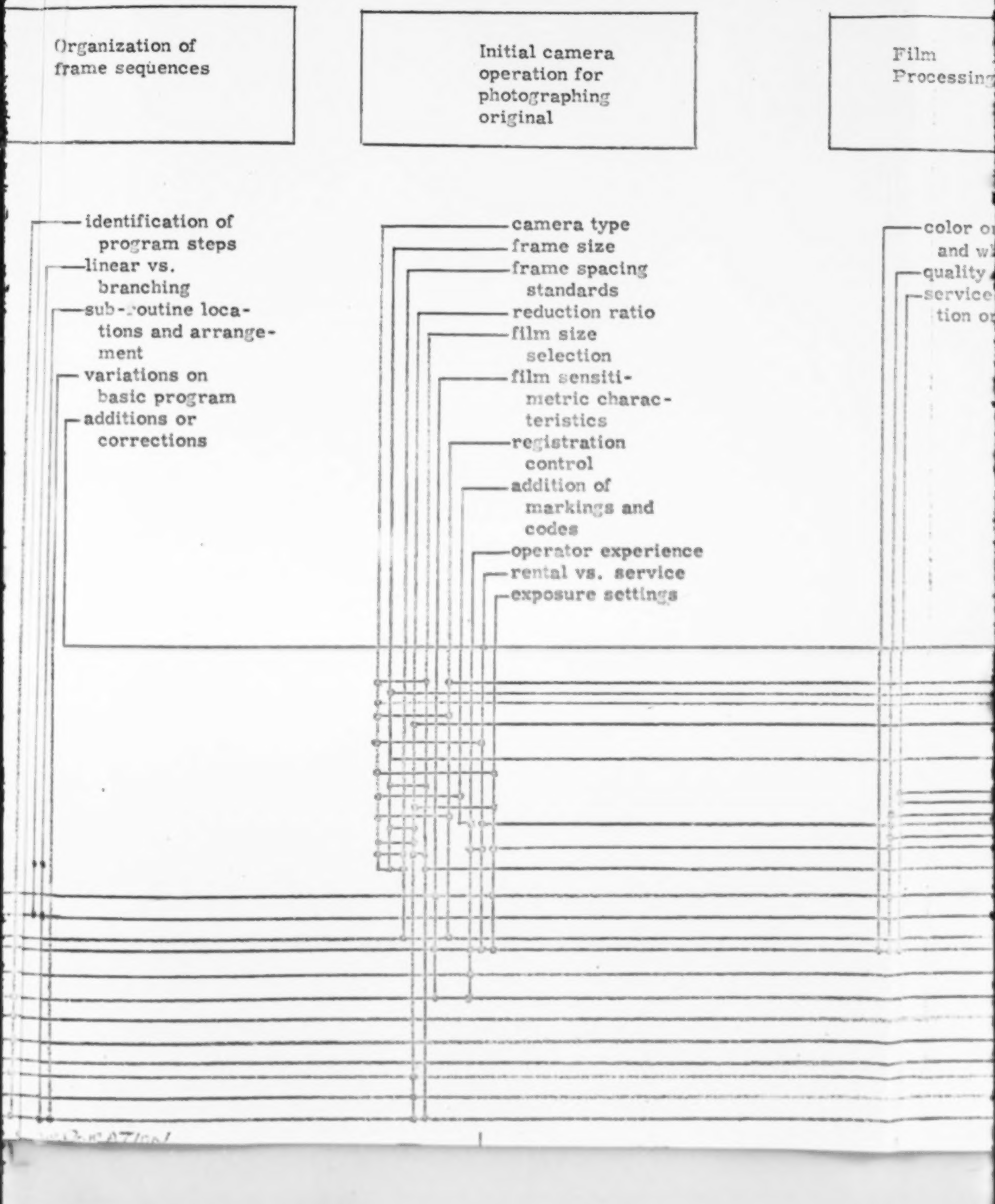
We would only add that in addition to the more important question of self-instructional material evaluation, an evaluation of the machine for its presentation should also be made. Unfortunately, the educators who have some first-hand basis for evaluation of instructional materials may have no basis for evaluating machine design. There is a sore need of information on machine characteristics to be presented on an impartial basis.



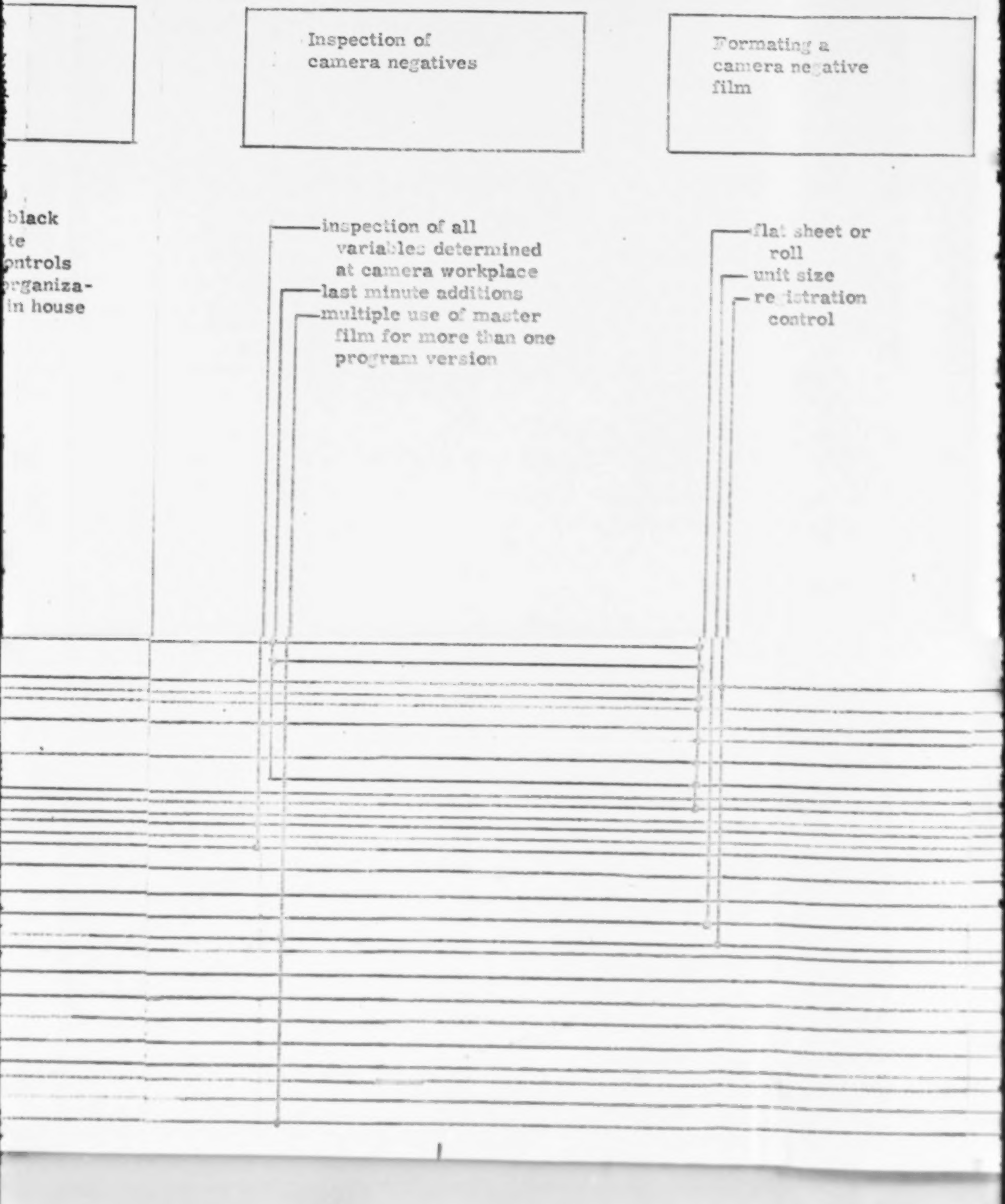
Preparation of original full-size copy

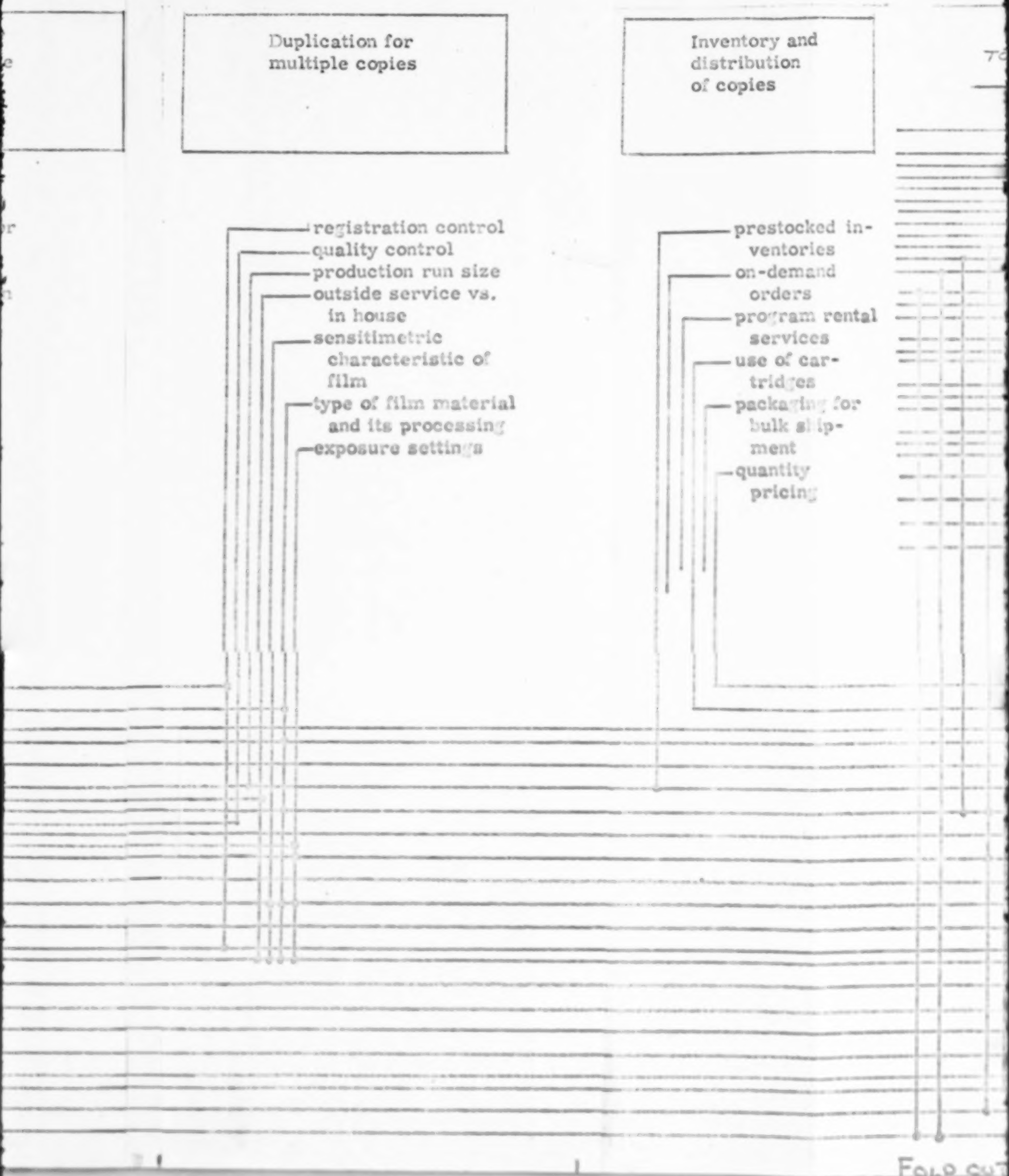
- frame layouts
- type style
- type size
- composition machine
- length of line
- number of lines
- variety of art-work (sheet sizes)
- use of existing graphics through photocopy
- color processes
- registration control
- programming format
- continuous tones vs. line drawings

INFORMATION DYNAMICS

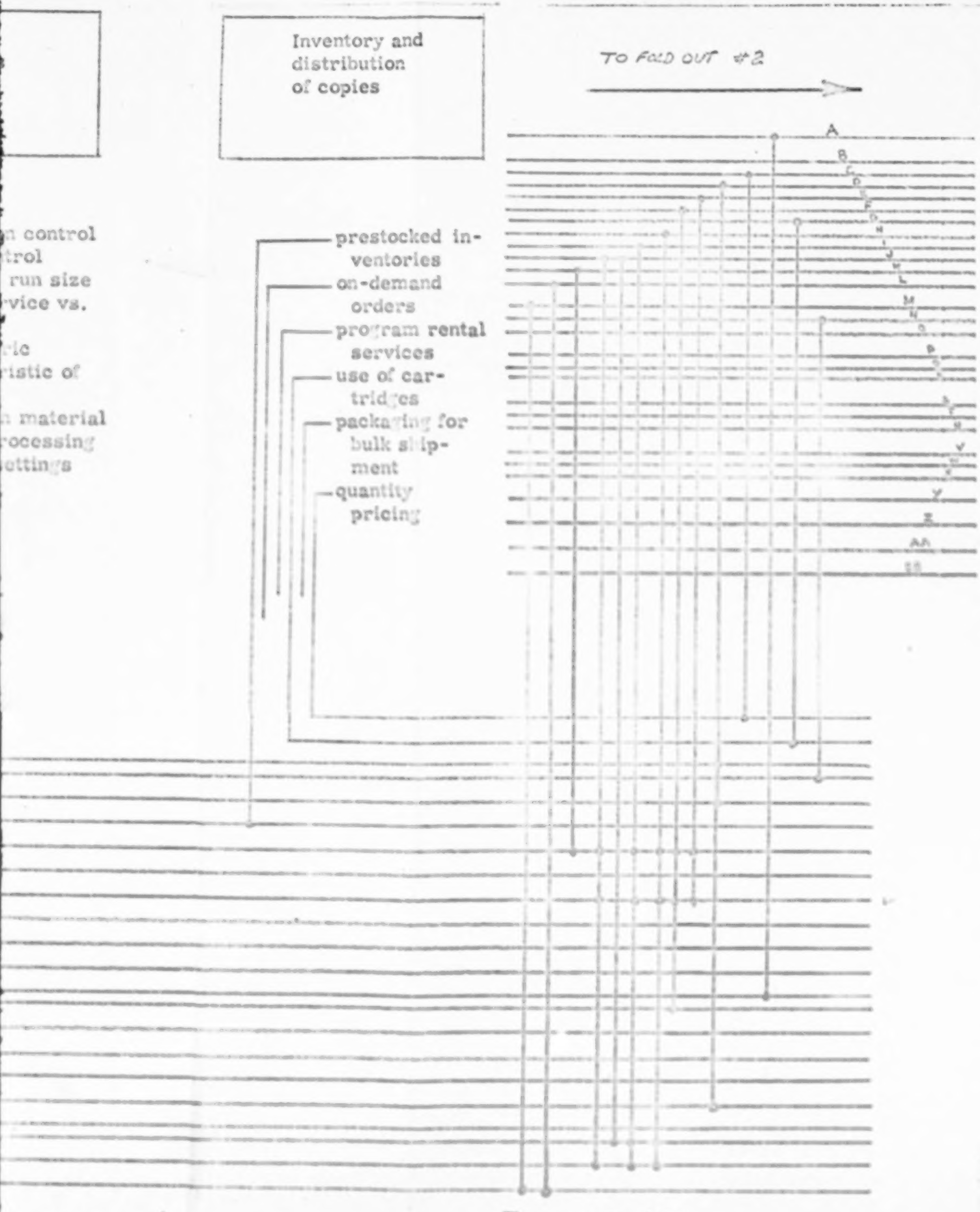

Organization of frame sequences
identification of program steps
linear vs. branching
sub-routine locations and arrangement
variations on basic program
additions or corrections
Initial camera operation for photographing original
camera type
frame size
frame spacing standards
reduction ratio
film size selection
film sensitimetric characteristics
registration control
addition of markings and codes
operator experience
rental vs. service
exposure settings
Film Processin
color o
and w
quality
service
tion o

## PROGRAM FILM PUBLICATION

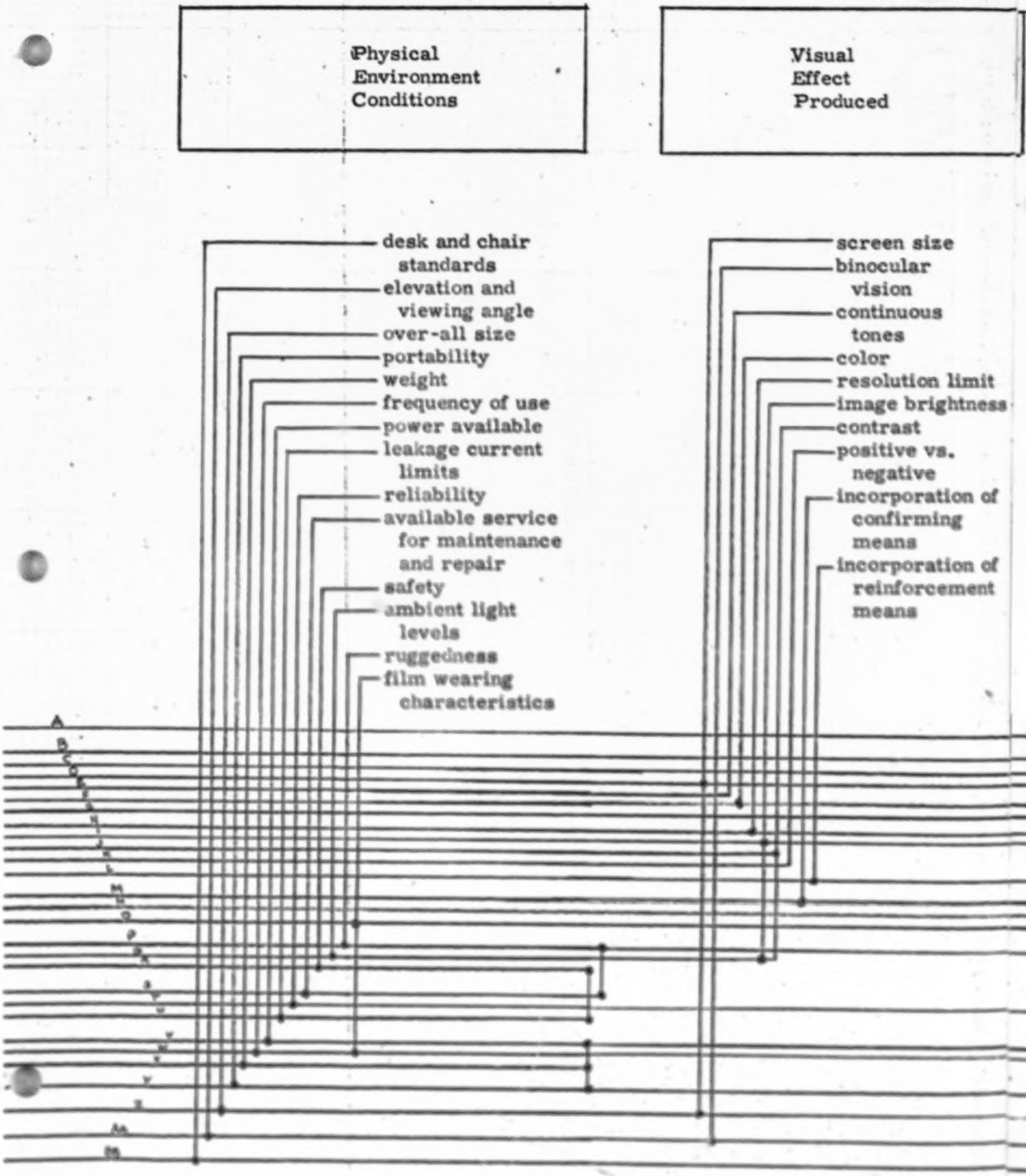

Inspection of camera negatives
Formating a camera negative film
black
te
ontrols
rganiza-
in house
inspection of all variables determined at camera workplace
last minute additions
multiple use of master film for more than one program version
flat sheet or roll
unit size
registration control


Duplication for multiple copies
registration control
quality control
production run size
outside service vs. in house
sensitimetric characteristic of film
type of film material and its processing
exposure settings
Inventory and distribution of copies
prestocked inventories
on-demand orders
program rental services
use of cartridges
packaging for bulk shipment
quantity pricing

FOLD OU


n control
trol
run size
rvice vs.
ric
ristic of
n material
rocessing
ettings
Inventory and distribution of copies
prestocked inventories
on-demand orders
program rental services
use of cartridges
packaging for bulk shipment
quantity pricing
TO FOLD OUT #2
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
AA
BB

FOLD OUT # I


Physical Environment Conditions
Visual Effect Produced
desk and chair standards
elevation and viewing angle
over-all size
portability
weight
frequency of use
power available
leakage current limits
reliability
available service for maintenance and repair
safety
ambient light levels
ruggedness
film wearing characteristics
screen size
binocular vision
continuous tones
color
resolution limit
image brightness
contrast
positive vs. negative
incorporation of confirming means
incorporation of reinforcement means
A
B
C
Z
AA
BB

DETAILED EQ

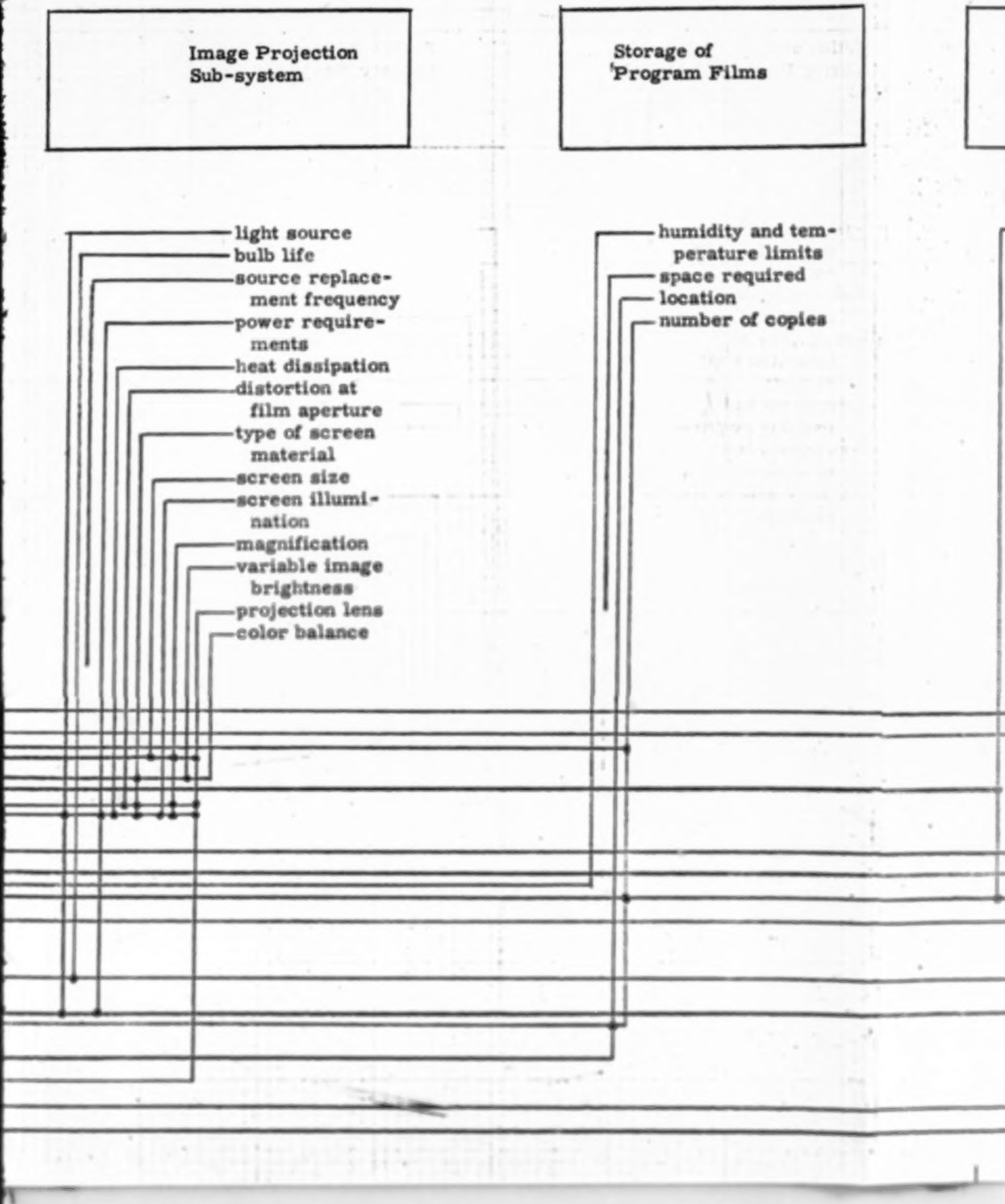
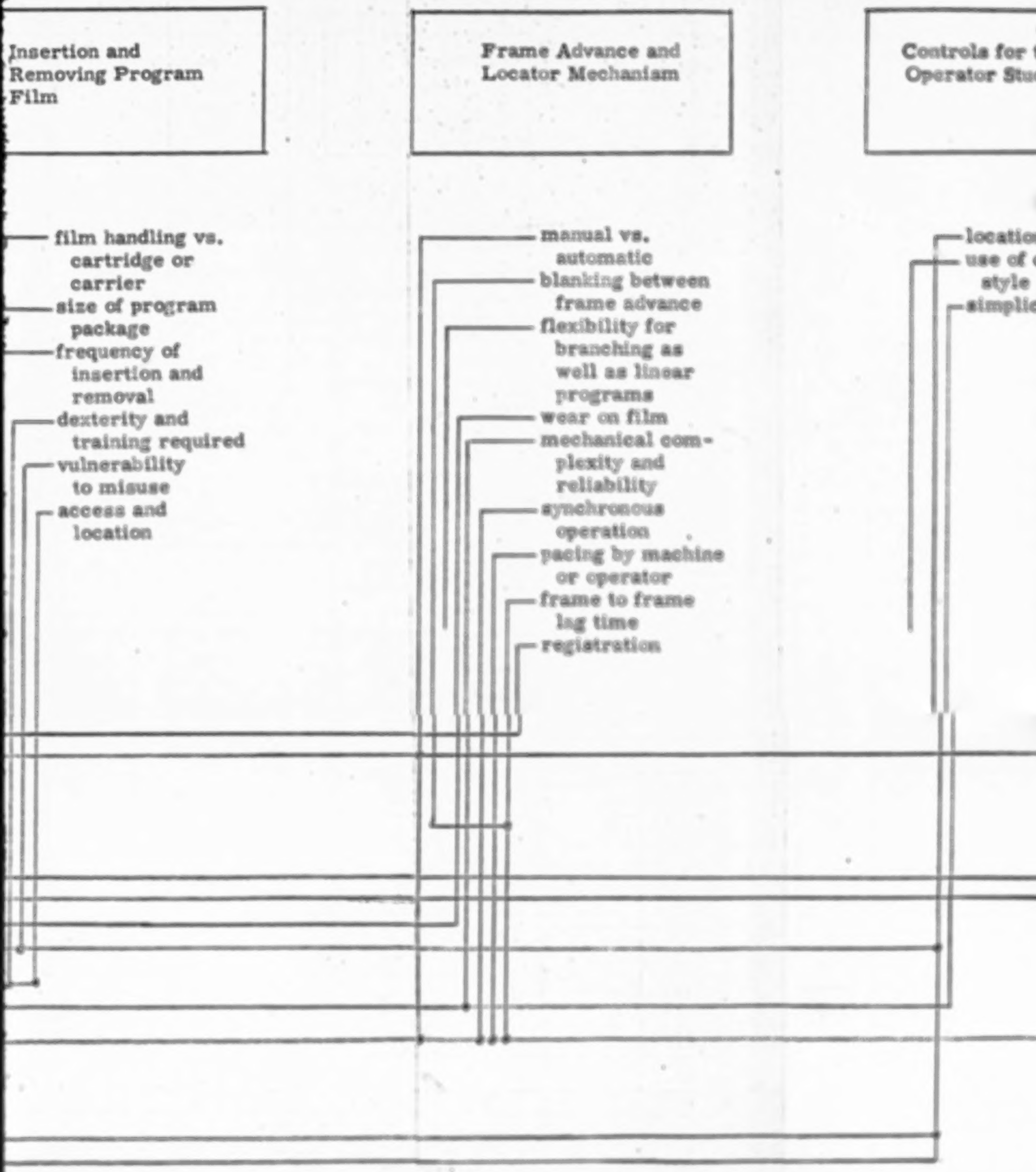
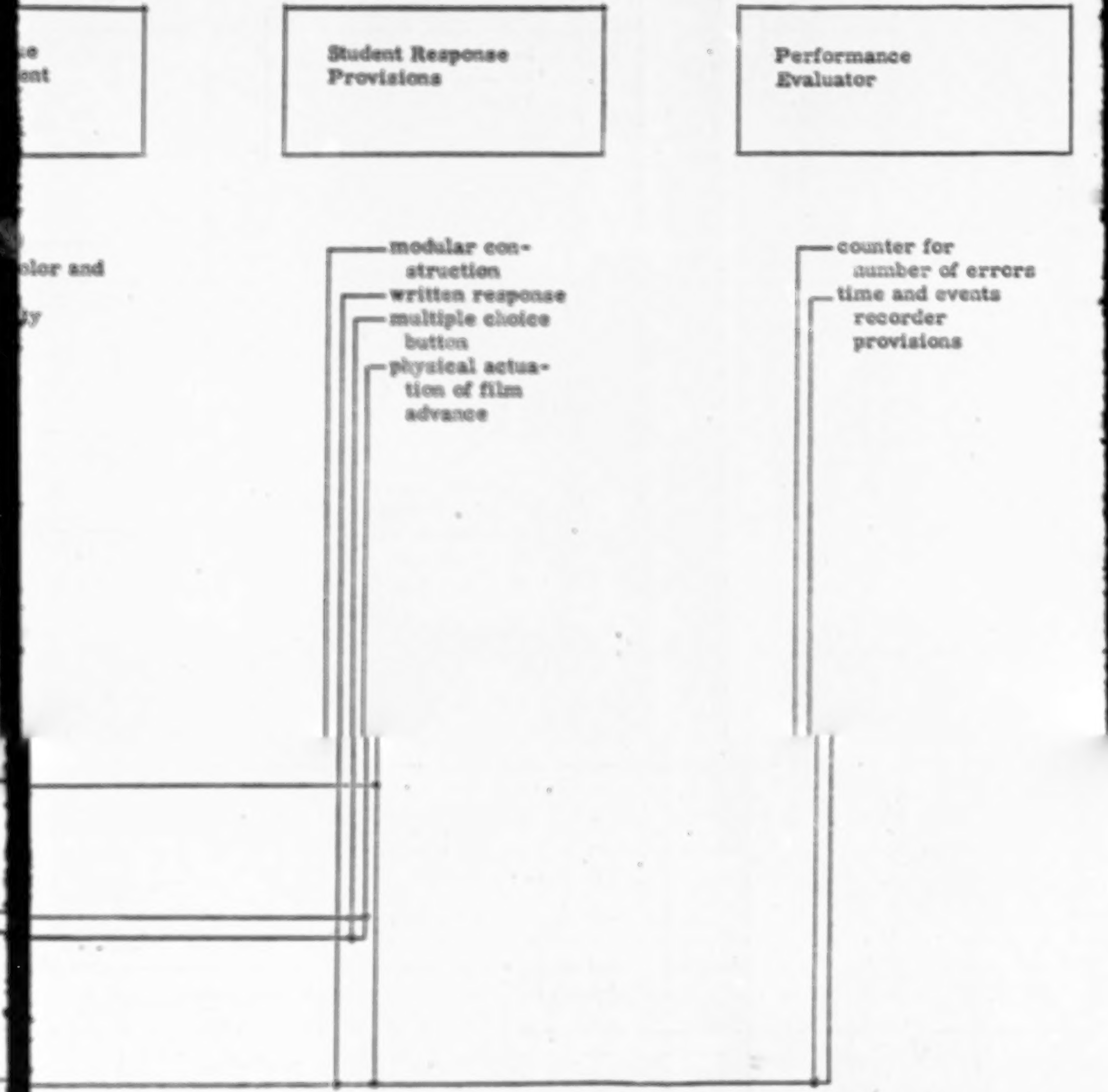


Image Projection Sub-system
Storage of Program Films
light source
bulb life
source replacement frequency
power requirements
heat dissipation
distortion at film aperture
type of screen material
screen size
screen illumination
magnification
variable image brightness
projection lens
color balance
humidity and temperature limits
space required
location
number of copies

PMENT DESIGN CONSIDERATIONS


Insertion and Removing Program Film
film handling vs. cartridge or carrier
size of program package
frequency of insertion and removal
dexterity and training required
vulnerability to misuse
access and location
Frame Advance and Locator Mechanism
manual vs. automatic
blanking between frame advance
flexibility for branching as well as linear programs
wear on film
mechanical complexity and reliability
synchronous operation
pacing by machine or operator
frame to frame lag time
registration
Controls for
Operator Stu
locatio
use of
style
simplic


ce
ent
Student Response Provisions
Performance Evaluator
olor and
ly
modular construction
written response
multiple choice button
physical actuation of film advance
counter for number of errors
time and events recorder provisions

FOLDOUT I

Instructional Diagra

# LIST OF REFERENCES

1. Carr, William J. *Self-Instructional Devices: A Review of Current Concepts.* Bucknell University, August 1959. OTS PB-161449.

2. Lumsdaine, A. A. and Robert Glaser, ed. *Teaching Machines and Programmed Learning.* National Education Association, Department of Audio-Visual Instruction, Washington, D. C., 1960.

3. Galanter, Eugene. *Automatic Teaching: The State of the Art.* John Wiley & Sons, Inc., New York, N. Y., 1959.

4. Crowder, Norman A. "Simple Ways to Use the Student Response for Program Control," page 286 of Reference 2.

5. Stolurow, L. M. *Teaching by Machine.* U. S. Department of Health, Education and Welfare, Washington, D. C., 1962. Cooperative Research Monograph No. 6, OE-34010.

6. Woodson, Wesley E. *Human Engineering Guide.* University of California Press, Berkeley, California, 1960.

7. Carmichael, Leonard and Walter F. Dearborn. *Reading and Visual Fatigue.* Houghton Mifflin & Company, Riverside Press, Cambridge, Massachusetts, 1947.

8. Sears, Francis S. *Optics.* Addison-Wesley Company, Inc., Reading, Massachusetts, 1949.

9. Lewis, Chester M. and William H. Offenhauser. *Microrecording: Industrial and Library Applications.* Interscience Publishers, New York, 1956.

10. Berul, Lawrence H. *Automated Composing Systems and Techniques: A State of the Art Report.* Information Dynamics Corporation, Wakefield, Massachusetts, January 1963.

11. Silverman, R. E. *Automated Teaching: A Review of Theory and Research.* U. S. Navy Training Device Center, Port Washington, L. I., N. Y. Technical Report 507-2; ASTIA Doc. No. AD-241283.

12. Lysought, Jerome P. "Programmed Learning and Teaching Machines Industrial Training," *The Journal of the American Society of Training Directors,* June 1961.

13. Mayer, Sylvia R. *The Effect of Induced Effort, Tracing and Drawing on Visual Form Learning.* Boston University Optical Research Laboratory, September 1954. Technical Note 115.

14. American Standards Association, Inc., 70 East 45th Street, New York 17, N. Y. C70.1 - 1953.

15. Carmichael, Leonard and Walter F. Dearborn. Reading and Visual Fatigue. Houghton Mifflin & Company, Riverside Press, Cambridge, Massachusetts, p. 145.

16. Ibid., page 58.

17. Johnson, B. K. Optics and Optical Instruments. Dover Publications, Inc., New York, 1960.

18. Eshbach, Ovid W. Handbook of Engineering Fundamentals. Wiley and Sons, New York, 1947, p. 9-33.

19. Ullrich, O. A., J. R. Stock, J. M. Dugan, and L. E. Walkup. Design Improvements for Simple Microimage Viewers: Report to Council on Library Resources, Washington, D. C. Batelle Memorial Institute, Columbus, Ohio, 1961.

20. Whitaker, G. C. "The Role of Microfilm in the Teaching Machine Field," Volume XI, Proceedings 1962 Convention of National Microfilm Association. National Microfilm Association, Annapolis, Maryland, 1962.

21. Silverman, R. E. Response Mode, Pacing, and Motivational Effects in Teaching Machines. U. S. Navy Training Device Center, Port Washington, L. I., N. Y. ASTIA, Doc. No. AD-262779.

22. The Hercules Model Presto Splicer. Prestoseal Manufacturing Corporation, 37-27 33rd Street, Long Island City 1, New York, 1961.

# APPENDIX A

## INSTRUCTIONS FOR THE USE OF THE NATIONAL BUREAU OF STANDARDS MICROCOPY RESOLUTION TEST CHART

In general, the reduction in dimensions of documents by the use of a microcopying camera is limited by the resolving power of the lens, the resolving power of the photographic material, inaccuracy of focusing, vibration, and systematic relative motion of the optical image with respect to the photographic material. In order to evaluate the performance of microcopying systems, the resolution test chart was prepared. This chart consists of a series of black parallel-line patterns on a white background, the lines and spaces being of equal width. Each pattern contains two groups of 5 lines, at right angles to each other. The patterns range in spacial frequency from one to ten lines per millimeter. (It is conventional in photography to consider a line and the adjoining space together as "one line".) Three of these charts are placed on the copy board of the camera so that one chart is in the center of the field, one at the center of the long side, and another at the corner. The last two charts are oriented so that one group of lines of the patterns is radial with respect to the center of the field and the other group of lines is tangential.

The array of charts is then photographed in the same manner as documents. To test the focus of the lens a series of photographs is made with small changes in focus, keeping a record of focusing scale settings for future reference. After the films have been developed, the negatives are examined with a microscope. The magnification should be from 1/3 to 1 times the number of lines per millimeter to be observed. For example, to view 100 lines per millimeter, the microscope magnification should be between 30 and 100. The number of lines per millimeter, in the smallest pattern in which the lines can be counted with certainty, multiplied by the reduction ratio, gives the resolving power of the system. For example, if the smallest pattern resolved has 5.0 lines per millimeter and the reduction ratio is 19, the resolving power is 95 lines per millimeter.

In the center of the field the resolution should be independent of the orientation of the patterns. Away from the center of the field, it is usually found that the resolution for radial and tangential patterns is not the same and, furthermore, changes with the distance. This effect is caused by aberrations of the lens. If, in the center of the field, the resolution of lines in one direction is less than for those in another direction, it is likely that the camera was vibrating during exposure or that there was relative motion of the image with respect to the film. In order to check the camera for vibration, make several exposures with the chart in the center of the field in one orientation, then turn it through an angle of 45° and make several more exposures. If the lines in both directions at both orientations are equally resolved, the camera is free from vibration.

In cameras of the type in which the document being copied and the film move during exposure, there is the possibility that the motions may be improperly matched so that the optical image moves with respect to the film. This can be checked by photographing resolution charts at the center of the field. If the camera is properly adjusted, the lines parallel to the direction of motion and perpendicular to it will be equally resolved. Otherwise the lines parallel to the direction of motion will show a higher resolution than those perpendicular.

The lowest resolution in any part of the picture area, the size and style of type, contrast, color of paper and ink, and the reduction ratio are factors determining the quality of reproduction on microcopying film. Three grades of reproduction, sufficient for the needs of practical microcopying, have been defined. They are:

1. Excellent copy - details of type are clearly defined.
2. Legible copy - readable without difficulty although serifs and fine details of type are not clearly defined.
3. Decipherable copy - readable with difficulty, letters e, c, and o are partly closed.

A correlation of these factors for commonly used styles of type has been established for these three grades of reproduction as follows:

1. For excellent copy, the height of the lower case letter e, as recorded on the film, should not be less than the distance across eight lines of the smallest resolution pattern resolved by the system.

2. For legible copy, the height of the e should be not less than the distance across five lines of the smallest pattern resolved.

3. For decipherable copy, the height of the e should be not less than the distance across three lines of the smallest pattern resolved.

For example, if it is desired to copy a newspaper, on which the height of the smallest e is 1.5 millimeters, at a reduction ratio of 19x, the minimum resolution required for the three grades of reproduction are:

1. For excellent copy, 101 lines per millimeter.
2. For legible copy, 63 lines per millimeter.
3. For decipherable copy, 38 lines per millimeter.

It is recommended that several photographs of the resolution charts be made on every roll of microcopy film used, preferably at both ends of the roll. This practice provides a convenient means of checking the operation and performance of the microcopying equipment, and provides a record which can be referred to later.

4th Edition
October, 1960

Photographic Research Section
National Bureau of Standards
U. S. Department of Commerce

USCOMM-NBS-DC

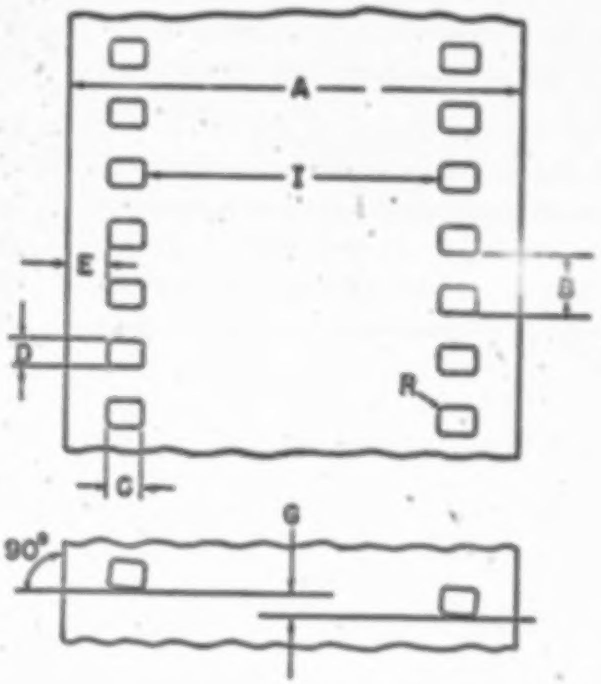
American Standard

# Dimensions for 35mm Motion-Picture Positive Raw Stock

Reg. U.S. Pat. Office

PH22.36-1954
Revision of Z22.36-1947

*UDC 778.5771.5



| Dimensions | Inches | Millimeters |
|---|---|---|
| A | 1.377 ± 0.001 | 34.98 ± 0.03 |
| B | 0.1870 ± 0.0005 | 4.750 ± 0.013 |
| C | 0.1100 ± 0.0004 | 2.794 ± 0.01 |
| D | 0.0780 ± 0.0004 | 1.98 ± 0.01 |
| E | 0.079 ± 0.002 | 2.01 ± 0.05 |
| *G | Not > 0.001 | Not > 0.025 |
| I | 0.999 ± 0.002 | 25.37 ± 0.05 |
| †L | 18.70 ± 0.015 | 474.98 ± 0.38 |
| R | 0.020 | 0.51 |

These dimensions apply to the film immediately after cutting and perforating. This film is used for motion-picture prints and sound recording.

* Method of indicating G is the main change from Z22.36–1947.

† This dimension represents the length of any 100 consecutive perforation intervals.

Approved April 5, 1954, by the American Standards Association, Incorporated
Sponsor: Society of Motion Picture and Television Engineers

*Universal Decimal Classification

Copyright 1954 by the American Standards Association, Incorporated

10 East 40th Street, New York 16, N.Y.
Printed in U.S.A.
ASA½M554

## Appendix

(This Appendix is not a part of the foregoing Standard.)

The dimensions given in this standard represent the practice of film manufacturers in that the dimensions are for film immediately after perforation. The punches and dies themselves are made to tolerances considerably smaller than those given, but owing to the fact that film is a plastic material, the dimensions of the slit and perforated film never agree exactly with the dimensions of the punches and dies. Shrinkage of the film, due to change in moisture content or loss of residual solvents, invariably results in a change in these dimensions during the life of the film. This change is generally uniform throughout the film.

The uniformity of perforation is one of the most important of the variables affecting steadiness of projection.

Variations in pitch from roll to roll are of little significance compared to variations from one sprocket hole to the next. Actually, it is the maximum variation from one sprocket hole to the next within any small group that is important.

# APPENDIX C

## An Interim Report on the Use of Graphics in High School Textbooks

This report contains the results of a content analysis of all second year algebra textbooks approved by the Boston School Committee for use in public schools. A list of these books appears in the Appendix. For the purposes of this study these should be a reasonable representation of second year algebra books in general.

The books were examined once to identify stimulus categories and once to identify response categories. The procedure followed was this:

a. Examination for Stimuli. Using books as strata, a proportional stratified sample of book pages was drawn. The sample size was chosen to correspond to the average number of pages per book. A high school teacher was employed to examine each of the pages so drawn, and to record on a card any graphics on the page. The operational definitions used for graphic stimuli and responses will be found in the Appendix. For each graphic stimulus so recorded, the examiner was directed to also examine adjacent material and record what the author of the text asked the student to do with the graphic stimulus, i.e., what response was demanded. No attempt was made to categorize non-graphic stimuli, but the response was categorized whether or not it involved graphic activity.

b. Examination for Responses. Since it was desired to record only specific unequivocal demands for responses, an arbitrary decision was made to sample only exercises at the end of each chapter. Ten per cent of the questions in these exercises were selected in a proportional stratified sample, where strata were chapter exercises. This percentage was chosen to make the number of responses sampled roughly correspond to the number of stimuli obtained in the preceding examination. For each response thus recorded, the corresponding stimulus (the form of the exercise question) was also noted. Since the questions themselves were the sampling units, rather than book pages, it was possible to categorize each stimulus and response, regardless of whether graphic or non-graphic.

Once cards with stimuli and responses recorded on them had been obtained, the next step was to establish stimulus and response categories. This was done by giving each of two judges a small sample of the cards and requesting that they sort them into homogeneous groups according to the stimuli which they described and then the responses which they described.

This sort was performed independently by the two judges. They subsequently compared the results of their sorts, collated their groupings, and agreed upon a title for each category. With tentative categories thus established, the judges independently sorted the entire sample. This sort verified the categories and served as a reliability check. The nature of distribution precludes statistical assessment of the degree of reliability at this point, but out of 473 stimulus-response pairs, there was complete agreement on categorization of 415 stimuli and 446 responses. The cards were then re-examined by the judges, disagreements were resolved by discussion, and the accompanying table was prepared showing frequency of occurrence of stimulus-response combinations. No attempt to interpret these findings will be made at present. The remainder of this report will be a listing of the obtained categories with illustrative examples.

## Stimulus Categories

1. Arrows used to direct attention (either in black and white, or in color).

   Example: Red arrows are superimposed on the solution of an equation requiring division of a polynomial by a monomial. They call attention to the dividend and the quotient so that the student may recognize that they have the same number of terms.

2. Line drawings, diagrams, and sketches (either in color or black and white).

   Example: A line drawing of a desk calculating machine with grey and red shading is presented as an example of one of a variety of computer types discussed in the text.

3. Photographs (black and white only).

   Example: A photograph of a baseball diamond used to add interest to a discussion of probability.

4. Drawings and photographs superimposed or combined.

   Example: Photograph of engineers examining a mecked marked-up aircraft with drawings of slide rule and drafting instruments forming a background. Used to call attention to importance of mathematics in engineering.

5. Color used to emphasize a word or number.

   Example: If $x = 0$, then $2x^2 - 5x + 4 = 4$ where "if" and "then" are printed in red.

6. Lines used to illustrate operations to be performed (either in color, or black and white).

Example: In solving a probability problem, where factorials are represented in both a numerator and denominator, red lines are used to cross out numbers to illustrate the cancellations which may be performed.

7. Box or background used for emphasis (color or black and white).

Example: Rules for solving an incomplete quadratic equation set off in a red box.

8. Graphs (in color or black and white).

Example: Demonstration of graphic solution of a quadratic with the parabola in black and the coordinate system printed in red.

9. Geometric figures (color or black and white).

Example: A rectangle printed in black with the diagonal printed in red as an aid in solving a problem where the diagonal and one side are given and the other side is to be found.

10. Tables (both in color and black and white). (Determinants are also included in this category because of their similar structure)

Example: For a given value of x, the corresponding value of y is presented in a table. The table is framed in red, with the values printed in black.

11. Verbally stated problems.

Example: Write the formula for the sum of n terms of a geometric progression.

12. Numeric problems.

Example: Given an equation, prove it for given values.

13. Combined verbal and numeric problems.

Example: A man has twice as many dimes as he has pennies, and he has 3 fewer nickels than dimes. If he has $2.64 in all, how many coins of each kind does he have.

14. Verbal questions.

Example: What is meant by the term "polynomial?"

15. Numeric questions.

Example: $(ab^3/a^2 - ab)\ (a - b/ab)$

16. Combined verbal and numeric questions.

Example: What is the y-intercept of $y = 2x - 10$?

17. Numeric questions with verbal instructions.

Example: Remove symbols of grouping and combine like terms: $10 - (a - 1)$

18. Questions or problems with supplementary graphic materials.

Example: Diagram of a triangle with sides labeled and instructions to find the tangent of an indicated angle.

## Response Categories

1. Construct a table.

Example: Given an equation, in x and y, prepare a table showing y values for given values of x.

2. Draw a diagram.

Example: Draw the appropriate figures for showing that the square of a tangent from a point to a circle is equal to the product of a secant from the point and the external segment of the secant.

3. Draw a graph.

Example: Graph the equation $y = x^2 + 2x + 1$

4. Associate graphic and numeric stimuli.

Example: Given an equation and its graphic representation examine the two in order to be able to recognize the graph as "belonging to" the equation.

5. Associate graphic and verbal stimuli.

   Example: Given a graph of a line with points scattered about it, associate this with a verbal explanation of a least squares fit.

6. Solve a problem numerically.

   Example: If y varies inversely as x, and y = 50 when x = 10, what is the value of y when x = 20?

7. Perform numeric operations.

   Example: Perform operations like divide, factor, take square root, etc.

8. Follow demonstration of a method.

   Example: Given an illustration of synthetic division, observe the steps so that the operations could be performed if required.

9. Solve an equation.

   Example: Given an equation, solve for any desired values.

10. Perform derivation or proof.

    Example: Prove the theorem that in any proportion the product of the means is equal to the product of the extremes.

11. Attend to an emphasized objective.

    Example: Make use of contextual cues to abstract the important elements from a page, section, etc.

12. Take interest in a text objective.

    Example: Given an illustration of a space ship, exhibit an increased motivation for working the mathematical problems adjacent to it.

13. Answer verbally.

    Example: Answer in words the question, "What is an antilogarithm?"

14. Insufficient information to establish a category. Those stimulus categories where the response proved too nebulous for categorization.

# STIMULUS AND RESPONSE CATEGORIES — ALGEBRA

ENTRIES IN THE TABLE ARE FREQUENCIES WITH WHICH THE CATEGORIES WERE ENCOUNTERED IN A RANDOM SAMPLING OF SEVEN BOOKS

STIMULI

| RESPONSES | | 1a ARROWS USED TO DIRECT ATTENTION COLOR | 1b BL. & WH. | 2a LINE DRAWINGS AND DIAGRAMS COLOR | 2b BL. & WH. | 3 BL. & WH. PHOTOGRAPHS | 4 BL. & WH. DRAWING SUPERIMPOSED ON PHOTOGRAPH | 5 COLOR USED TO EMPHASIZE A WORD OR NUMBER | 6a LINES USED TO ILLUSTRATE OPERATIONS TO BE PERFORMED COLOR | 6b BL. & WH. | 7a BOX OR BACKGROUND USED FOR EMPHASIS COLOR | 7b BL. & WH. | 8a GRAPHS COLOR | 8b BL. & WH. | 9a GEOMETRIC FIGURES COLOR | 9b BL. & WH. | 10a TABLES COLOR | 10b BL. & WH. | 11 VERBALLY STATED PROBLEMS | 12 NUMERIC PROBLEMS | 13 COMB. VERBAL-NUMERIC PROBLEMS | 14 VERBAL QUESTIONS | 15 NUMERIC QUESTIONS | 16 COMB. VERBAL-NUMERIC QUESTIONS | 17 NUMERIC QUESTIONS WITH VERBAL INSTRUCTIONS | 18 QUESTIONS OR PROBLEMS WITH SUPPLEMENTARY GRAPHIC | PER CENT OF TOTAL (of responses) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| INSUFFICIENT INFORMATION | 14 | | | | | | | | | | | | | | | | | | 2 | | 2 | 3 | 1 | 13 | | | 4 |
| ANSWER VERBALLY | 13 | | | | | | | | | | | | | | | | | 1 | | | | 6 | | 1 | | | 2 |
| TAKE INTEREST IN TEXT OBJECTIVE | 12 | 2 | | 2 | | 2 | 1 | | | | | | | | | | 1 | | | | | | | | | | 2 |
| ATTEND TO AN EMPHASIZED OBJECTIVE | 11 | | | | | | | 8 | 1 | | 2 | 2 | | 1 | | 1 | 3 | 3 | | | | | | | | | 4 |
| PERFORM DERIVATION OR PROOF | 10 | | | | | | | | | | | | | | | | | | | 1 | 1 | 2 | 3 | 3 | 3 | | 3 |
| SOLVE EQUATION | 9 | | | | | | | | | | | | | | | | | | | 3 | 2 | | 10 | 7 | 11 | | 7 |
| FOLLOW DEMONSTRATION OF METHOD | 8 | 2 | 1 | | | 2 | | 8 | 6 | 2 | 7 | | 2 | 1 | | | 2 | 3 | | | | | | 1 | | | 8 |
| PERFORM NUMERIC OPERATIONS | 7 | | | | | | | | | 2 | | | | | | | | | 2 | | 4 | 5 | 58 | 41 | 34 | | 31 |
| SOLVE A PROBLEM NUMERICALLY | 6 | | | | | | | | | | | | | | | 1 | 1 | | 8 | | 19 | | 5 | 8 | 2 | 1 | 10 |
| ASSOC. GRAPHIC & VERBAL STIMULI | 5 | 1 | | 4 | 2 | 2 | | | 1 | | 3 | | 24 | 13 | 15 | 10 | 8 | 4 | | | | | | | | | 18 |
| ASSOC. GRAPHIC & NUMERIC STIMULI | 4 | 1 | | | | | | 1 | | 1 | 1 | | 6 | 1 | 5 | 1 | | | 2 | | | | | | | 2 | 4 |
| DRAW A GRAPH | 3 | | | | | | | | | | | | 3 | 1 | | 1 | | | 1 | | 4 | | 1 | 8 | 2 | | 4 |
| DRAW A DIAGRAM | 2 | | | | | | | | | | | | | | 1 | | | | | | 3 | | | 5 | | 1 | 2 |
| CONSTRUCT A TABLE | 1 | | | | | | | | | | | | | | | | | 1 | | | 2 | | | 1 | 1 | | 1 |
| PER CENT OF TOTAL (of stimuli) | | 1 | 0.4 | 1 | 0.6 | 1 | 0.4 | 4 | 2 | 1 | 3 | 0.6 | 7 | 4 | 4 | 3 | 3 | 3 | 3 | 1 | 8 | 3 | 18 | 18 | 11 | 1 | |

Texts: ✓Baker, et al, 1959 | Moon, et al. 1960 aut = Modern Biology
✓Dodge, 1959 | Smith + Lisonbee, 1958✓
✓Fitzpatrick + Bain, 1958

## Biology

| | Stimulus | S T% | Key | Response | S-R Pairs % | R T% |
|---|---|---|---|---|---|---|
| 2a | Line drawings in color 30 | 14.2 | 5 | Verbal R associating graphic with verbal S (problem, explanation, etc.) | 12.7 | |
| | | | 11 | Attend to emphasized Text objective | 0.5 | |
| | | | 12 | Become interested in topic | 0.7 | |
| | | | 13 | Answer a verbal question verbally. | 0.3 | |
| 2b | Line drawings in black and white 10 | 4.7 | 5 | | 4.7 | |
| 3a | Color Photograph 2.5 | 1.2 | 5 | | 1.2 | |
| 3b | Black and white photograph 47 | 22.2 | 5 | | 21.4 | |
| | | | 12 | | 0.5 | 1.2 |
| | | | 13 | | 0.3 | |
| 3c | Photo with enlarged detail 4.7 | 2.3 | 5 | | 2.3 | |
| 4a | Drawing and photograph combined - color .4 | 0.2 | 5 | | 0.2 | |
| 4b | Drawing and photograph combined - Black and White 1.0 | 0.5 | 5 | | 0.3 | |
| | | | 13 | | 0.2 | |
| 7a | Colored box or background 7.2 | 3.5 | 11 | | 3.3 | 3.8 |
| | | | 13 | | 0.2 | |
| | Black and White Graph .4 | 0.2 | 5 | | 0.2 | |

| Key | Stimulus | S T% | Key | Response | S-R Pairs % | R T% |
|---|---|---|---|---|---|---|
| 10a 10 | Table - color 6.0 | 3.0 | 5 | | 3.0 | |
| 10b | Table - Black and White 1.0 | 0.5 | 5 | | 0.5 | |
| 11 | Verbal Problem | 2.6 | 1 | Construct a Table (include matrix). | 0.2 | |
| | | | 5 | | 0.2 | |
| | | | 13 | | 1.7 | |
| | | | 15 | Use apparatus (tools) | 0.5 | |
| 14 | Verbal Question | 40.5 | 1 | | 0.5 | |
| | | | 2 | Draw a diagram or picture | 0.3 | 0.3 |
| | | | 5 | | 0.2 | |
| | | | 13 | | 39.2 | |
| | | | 15 | | 0.2 | 0.7 |
| 18 | Verbal - Numeric Problem or Question with supplementary graphic | 0.7 | 1 | | 0.2 | 0.9 |
| | | | 13 | | 0.5 | 42.5 |
| 19a | Sequence of photographs | 1.4 | 8 | Attend to demonstration of method. | 1.4 | 1.4 |
| 19b | Sequence of line drawings | 2.4 | 5 | | 2.4 | 49.1 |

# APPENDIX

## Operational Definition of a Graphic Stimulus

For the purposes of this study, a graphic stimulus is defined as any presentation of visual or auditory material which departs from a narrative format or differs from the dominant format of text. This will be interpreted to include pictures, graphs, music (whether written or aural), illustrative algebraic functions, logic diagrams, and tabular material. It will not include exposition in standard paragraph form, auditory presentation which can be readily put into written form (i.e., where all meaning is conveyed by written presentation), and mathematical symbols in conventional linear format.

## Operational Definition of a Graphic Response

For the purposes of this study, a graphic response is defined as behavior required of a student that involves vocalizing, drawing or writing that departs from narrative form or conventional linear format. This will be interpreted to include drawing pictures, graphs, and logic diagrams; vocalization in music, poetry, and foreign languages; and written tabular presentations. It will not include speech when the full meaning can be conveyed in written form, nor will it include manipulation of symbols in conventional linear format.

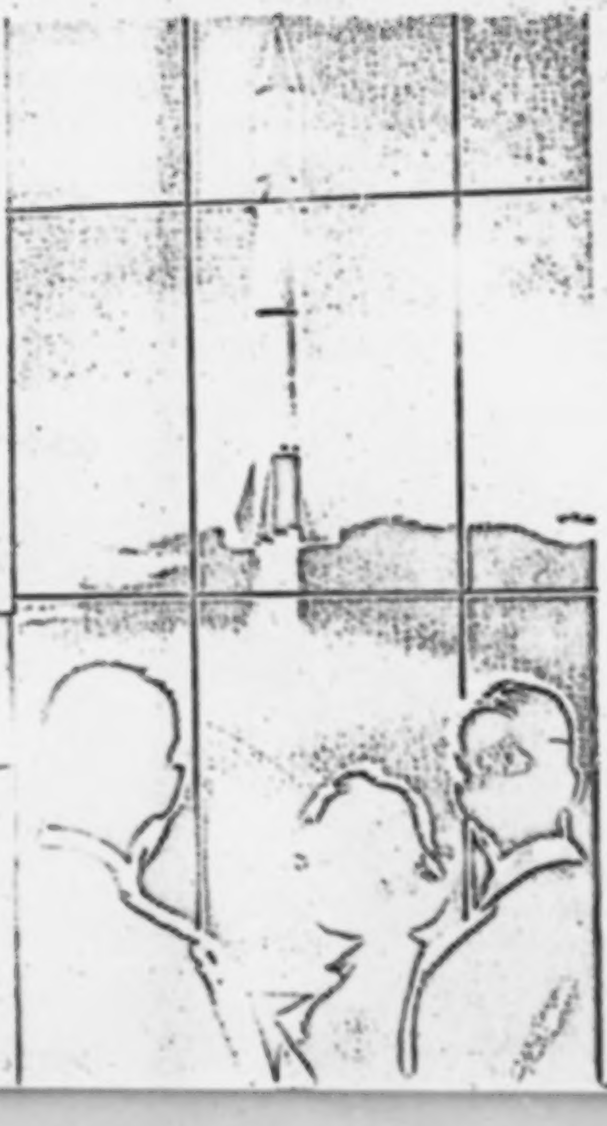
# APPENDIX D

(First Half)

## ALGEBRA

SAMPLES FROM

# Book Two



**Welchons**
**Krickenberger**
**Pearson**

**Example.** Add $3x^2-y^2+xy$, $2x^2+3xy-4y^2$, and $2y^2-6xy-x^2$.

**Solution.** The polynomials should be arranged according to the descending (or ascending) powers of one of the letters. In this solution they will be arranged in descending powers of $x$. We shall check the addition by letting $x=2$ and $y=3$.

$$\begin{array}{rcrl} \textbf{Solution} & & \textsc{check} & \\ 3x^2+\ xy-\ y^2 & = & 12+6-9= & 9 \\ 2x^2+3xy-4y^2 & = & 8+18-36= & -10 \\ -x^2-6xy+2y^2 & = & -4-36+18= & -22 \\ \hline 4x^2-2xy-3y^2 & & 16-12-27= & -23 \end{array} \Bigg\} = -23$$

RED COLOR

**Note.** Though theoretically any numbers could represent $x$ and $y$ in the check, most mathematicians hesitate to use 0 or 1. A mistake could be overlooked when 0 is substituted, since 0 times any number is 0. When 1 is used, an expression such as $3x^2$ has the same value as either $3x$ or 3.

[A] [illegible]

Find the sums:

1. $$\begin{array}{r} 2x^2-3xy+y^2 \\ -4x^2-6xy-y^2 \\ x^2+\ xy-y^2 \\ \hline \end{array}$$

2. $$\begin{array}{r} a-3b\ [illegible] \\ -7a\qquad -6c \\ 5b-\ c \\ \hline \end{array}$$

3. $x^3-4x^2-7x+6$, $2x^2-5x+2$, and $-5x^3+6x^2-8$
4. $x^3-y^3$, $2x^3-x^2y-2y^3$, $x^3-6y^3$, and $5x^2y-xy^2-y^3$
5. $m^2-mn+n^2$, $-$[illegible]$^2+m^2-3mn$, and $5m^2+n^2$
6. $4a^2-5b^2-$[illegible]$a$, $6a^2-5a$, and $7a^2+9b^2-11a$
7. $\frac{1}{3}x+\frac{3}{4}y-$[illegible]$z$, $\frac{1}{6}x+y-\frac{1}{4}z$, and $\frac{1}{2}x-\frac{3}{8}y-\frac{1}{16}z$
8. $\frac{1}{2}a+\frac{1}{3}b+\frac{1}{4}c$, $-\frac{3}{8}a-\frac{1}{5}b+\frac{1}{3}c$, and $a-b+c$
9. $0.7x^2-0.3y^2-0.7$, $1.2x^2-3.5y^2-0.3$, and $x^2+y^2-1$
10. $1.17m-2.15n+1.31p$ and $4.52m-5.65n-4.91p$

### Subtraction of Polynomials[A]

You know that 2 pounds and 7 ounces can be subtracted from 10 pounds and 11 ounces, giving 8 pounds and 4 ounces, as shown.

$$\begin{array}{r} 10 \text{ pounds} + 11 \text{ ounces} \\ \underline{2 \text{ pounds} + \ 7 \text{ ounces}} \\ 8 \text{ pounds} + \ 4 \text{ ounces} \end{array}$$

COLOR

ARROWS USED TO DIRECT ATTENTION

29. $x^a \cdot x^a$ 31. $x^{a+1} \cdot x^2$ 33. $2p^a \cdot 4p^a$

30. $h^{a+1} \cdot h^{a-1}$ 32. $-2x^{m+1} \cdot 3x^{1-m}$ 34. $7x^a(-3x^{1-a})$

## Division of Monomials [A]

Since $x^3 \cdot x^2 = x^5$, then $x^5 \div x^2 = x^3$.

Also $\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^3$.

This example illustrates the following law.

### Law of Exponents for Division

**The exponent of any number in a quotient is equal to its exponent in the dividend minus its exponent in the divisor.**

$$\frac{x^a}{x^b} = x^{a-b}$$

Let us divide $42x^3y^2z$ by $-7xyz$.

$$\frac{42x^3y^2z}{-7xyz} = \frac{42}{-7} \times \frac{x^3}{x} \times \frac{y^2}{y} \times \frac{z}{z} = -6x^2y\,1 = -6x^2y$$

### Rule for Dividing One Monomial by Another

1. **Find the quotient of the numerical coefficients to obtain the coefficient of the quotient.**
2. **Divide the literal factors of the dividend by the literal factors of the divisor to obtain the literal factors of the quotient.**

Example 1. Divide $-18a^4bc$ by $-3ac$.

Solution. $\frac{-18a^4bc}{-3ac} = 6a^3b$

Remember these facts.

1. Any number divided by itself equals 1.
2. Division by zero is impossible.

PROOF. Let $x$ and $y$ represent any two numbers except zero.

Suppose $\frac{x}{0} = y$

Then $y \times 0 = x$

But $y \times 0 = 0$

Then $x = 0$, which is impossible.

RED
COLORED

#1 COLOR

ARROWS USED TO DIRECT ATTENTION

7. $(x^2 - 2y)(x^2 - 5y)$
8. $(y^2 + 2y + 1)(y + 1)$
9. $(x - y)(x^2 + xy + y^2)$
10. $(a + b)(a^2 - ab + b^2)$
11. $(a^2 + a - 1)(a^2 - a + 1)$
12. $(m^2 + n^2 - mn)(m^2 + mn + n^2)$
13. $(x^2 - 5x + 4)(x^2 - x + 2)$
14. $(1 - 4x + x^2)(1 - x)$

15. Find the volume of a rectangular solid whose dimensions are $2a + 1$, $3a - 5$, and $a - 2$.

16. $(a - b)^3 = (a - b)(a - b)(a - b) = ?$

17. $(c - d)^2$
18. $(x + y)^3$
19. $(c^2 - d)^2$
20. $(a^n - b^n)(a^n + b^n)$
21. $(y^{2a} + y^a + 1)(y^a + 1)$
22. $(x^a + y^b + z^c)^2$

If $A = x^2 - xy + y^2$, $B = 3x - 4y$, and $C = 2x + y$, find the value of [A]

[illegible]

1. $A + B$
2. $C - B$
3. $2B + C$
4. $BC$
5. $AC$
6. $AB$
7. $A - BC$
8. $A + B - C$
9. $B^2 - C^2$

### Dividing Polynomials by Monomials [A]

**Rule for Dividing a Polynomial by a Monomial**
**Divide each term of the polynomial by the monomial.**

Example 1. Divide $5x^3y - 15x^2y^2 + 5xy$ by $5xy$.

Solution. $\frac{5x^3y - 15x^2y^2 + 5xy}{5xy} = x^2 - 3xy + 1$

Note that the quotient has the same number of terms as the dividend.

Example 2. $\frac{4x^{2m} - 6x^{m+2} - 8x^m}{2x^m} = 2x^m - 3x^2 - 4$

Divide: [A]

[illegible]

1. $(5a^2 - 20) \div 5$
2. $(18x - 27) \div (-9)$
3. $(8x - 12y) \div (-1)$
4. $(4c - 12d) \div (-4)$
5. $(6a^2 - 12b^2) \div 3$
6. $(x^4 - x^3 + x^2) \div x$
7. $(a^4 - a^3 - a) \div (-a)$
8. $(10x^3 - 12x^2 + 2x) \div (2x)$

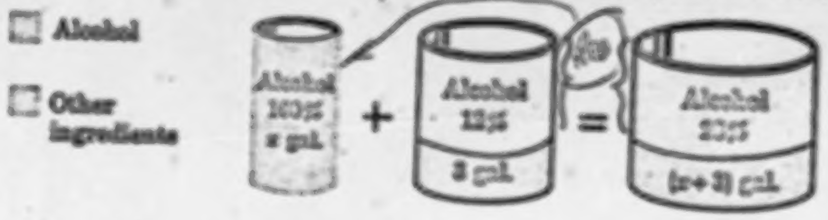
RED
COLOR

#1 COLOR

ARROWS USED TO DIRECT ATTENTION

**Example 2.** How much pure alcohol must be added to 3 gallons of a 12% solution to make a 28% solution?

**Solution.** Let $x$ = the number of gallons of pure alcohol required.



| | | | | | |
|---|---|---|---|---|---|
| Since | $x$ gallons of pure alcohol | + | alcohol in 3 gallons of a 12% solution | = | alcohol in $(x+3)$ gallons of a 28% solution |
| then | $1.00x$ | + | $.12(3)$ | = | $.28(x+3)$ |

$$\begin{aligned} M_{100} \qquad 100x + 12(3) &= 28(x+3) \\ 100x + 36 &= 28x + 84 \\ 72x &= 48 \\ x &= \tfrac{2}{3} \text{ gallons of pure alcohol to be added} \end{aligned}$$

PROOF. Left to student.

[illegible]

[A]

1. The doctor told Nurse Barton to dilute the antiseptic solution for the Jones baby from 25% to 15%. How much water must Miss Barton add to 1 pint of the 25% solution to get a 15% solution?

2. A garageman has 500 gallons of alcohol that is 75% pure. He wishes to add water until it is 40% pure. How much water must be added?

3. A grocer blends two grades of tea which are worth $1.20 and $1.60 a pound respectively. How much of each kind of tea must he use to make a mixture of 50 pounds which he can sell at $1.50 a pound?

4. How much cream which is 25% butterfat should be added to 1200 pounds of milk which is 3% butterfat to produce a milk testing 4% butterfat?

5. One alloy is 25% silver and another is 40% silver. How much of each should be used to produce 60 pounds of an alloy that is 30% silver?

#2A

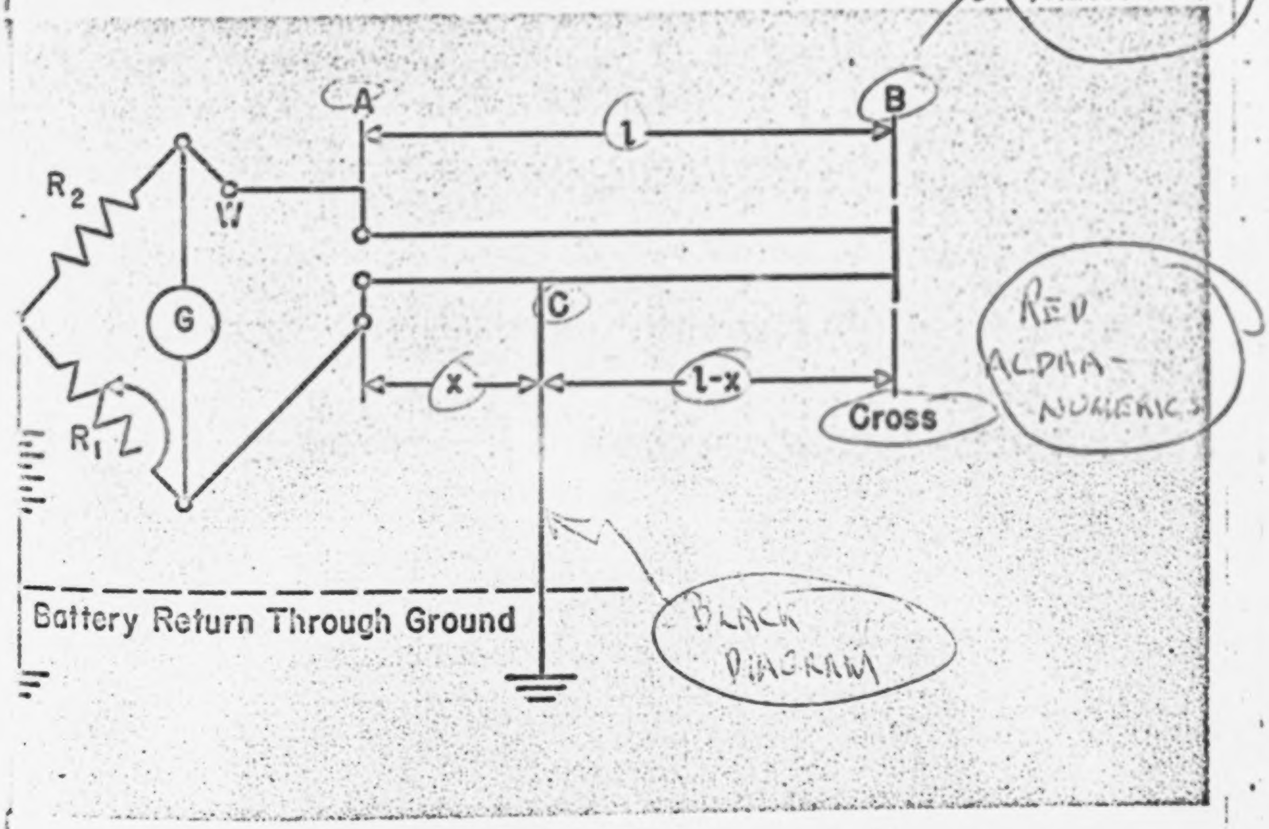
COLOR

LINE DRAWINGS AND DIAGRAMS

One method of locating a defect of this kind is the Murray Loop Test, a diagram of which is shown below. The formula $\frac{x}{2l - x} = \frac{R_1}{R_2}$ gives the relationship between the resistances $R_1$ and $R_2$, measured in ohms, and the distances $l$ and $x$, when the resistances of the two wires joining *A* and *B* are equal. The distance, represented by $x$ in the formula, from the point *A* to the defect in the wire can be determined by computation when the values of $l$, $R_1$, and $R_2$ are known.

In this case the engineer sent one of his men to make a cross of the wires at *B*. He connected a Wheatstone bridge to the two wires at *A*. He used 1000 ohms resistance for $R_2$ and adjusted the resistance $R_1$ to balance the bridge. The bridge balanced when $R_1$ was 146 ohms. By substituting known values in the formula given above, he obtained the equation $\frac{x}{120 - x} = \frac{146}{1000}$. Solving for $x$, he found that the defect was 15.29 miles from A.

Ordinarily the procedure is much more complicated. For example, the wires do not often have uniform resistance.



#2A

COLOR

LINE DRAWINGS AND DIAGRAMS

To help yourself remember these rules, think of positive numbers as gains and of negative numbers as losses; or remember how you add scores in certain games.

Examples. $(+4)+(-7)=-3$; $(-5)+(-3)=-8$
$(-10)+(-10)=-20$; $-7+7=0$

The numbers that are added are called addends. In the addition $5+8=13$ the addends are 5 and 8. The result of adding is called the sum.

REV COLOR

[A] EXERCISES

Add (or combine):

| | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| 1. | 4<br>−3 | −10<br>−4 | +8<br>3 | −11<br>3 | 4<br>−9 | 5<br>10 | −7<br>−4 |
| 2. | 6<br>−5 | 7<br>−3 | −4<br>−5 | −10<br>4 | −12<br>+12 | 10<br>0 | −11<br>1 |
| 3. | $\frac{1}{2}$<br>$-\frac{3}{2}$ | $\frac{2}{5}$<br>$-\frac{3}{5}$ | $-\frac{4}{7}$<br>$-\frac{3}{7}$ | $-\frac{2}{5}$<br>$+\frac{3}{5}$ | $\frac{3}{4}$<br>$-\frac{1}{4}$ | $-\frac{7}{8}$<br>$-\frac{3}{8}$ | $6\frac{1}{4}$<br>−7 |
| 4. | −1.0<br>−0.3 | 2.7<br>−1.5 | 7.6<br>2.4 | −8.1<br>4.1 | −10.0<br>−2.1 | −9.4<br>−2.6 | −7<br>3.4 |

Simplify:

5. $(+7)+(-6)$
6. $(-3)+(-20)$
7. $-8-9$
8. $-4-60$
9. $(-2)+(-15)$
10. $\frac{1}{2}+(-\frac{1}{3})$
11. $5.6+(-10)$
12. $-7.3+(-2.7)$

Remember that the commutative and associative laws hold for negative numbers. When you are to add several numbers, those laws make it possible for you to add the positive numbers first, to add the negative numbers next, and then to combine the two sums.

### Subtraction of Positive and Negative Numbers [A]

IN REV

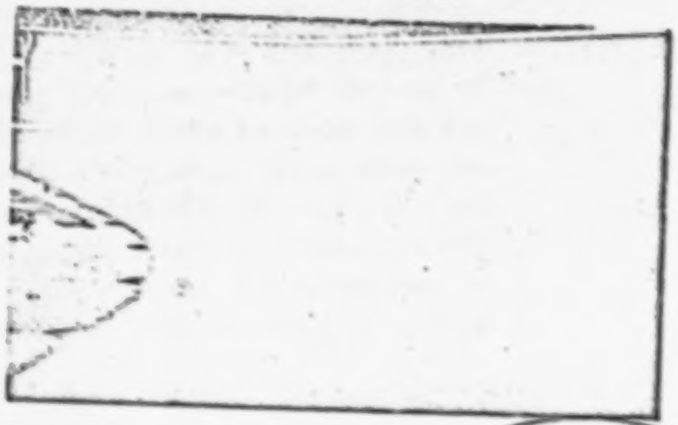
**Rule for Subtracting Positive and Negative Numbers**

**To subtract one number from another,**
**change the sign of the subtrahend and proceed as in addition.**

#5

COLOR

USED TO EMPHASIZE A WORD
OR NUMBER



To obtain some idea of the meaning of permutations and combinations, suppose that a high-school baseball coach has three boys, Al, Bill, and Chuck, each of whom can pitch or catch. Let us see how many batteries (a battery consists of a pitcher and catcher) can be had with these three players. Without reference to who pitches and who catches, three different batteries can be formed. They are:

| FIRST BATTERY | SECOND BATTERY | THIRD BATTERY |
|---|---|---|
| Al and Bill | Al and Chuck | Bill and Chuck |

THIS LINE IN RED

These three groups (batteries) are combinations of 3 boys taken 2 at a time ($_3C_2$). Each of these three batteries can be arranged in two ways as follows:

FIRST BATTERY

| | | | |
|---|---|---|---|
| Al | pitcher | Bill | pitcher |
| Bill | catcher | Al | catcher |

SECOND BATTERY

| | | | |
|---|---|---|---|
| Al | pitcher | Chuck | pitcher |
| Chuck | catcher | Al | catcher |

THIRD BATTERY

| | | | |
|---|---|---|---|
| Bill | pitcher | Chuck | pitcher |
| Chuck | catcher | Bill | catcher |

These six groups are permutations of 3 boys taken 2 at a time ($_3P_2$). Each of the three combinations above has two permutations. There is no order in combinations, but there is order in permutations.

A combination is a group of objects in which the order or arrangement is not considered.

UNDERLINES ARE RED!

A permutation is an arrangement of a group of objects in a definite order.

The discussion and problems of this chapter depend upon the following fundamental principle:

IN RED

**If one act can be performed in any one of *m* different ways,**
**and if after it has been done**
**a second act can be done in *n* different ways,**
**then the number of ways of performing the two acts in succession is *mn*.**

#5

COLOR

USED TO EMPHASIZE A WORD
OR NUMBER

In your first course in algebra you learned short methods of finding products such as $(x + 3\,y)(5\,x - 2\,y)$, $(a - 6)(a + 6)$, and $(m + 2)(m + 2)$, and could state the products without the use of pencil and paper. Products which can be found by short cuts are called special products. The products above are of three types, the second and third products being special types of the first. In the first part of this chapter you will study special products. Then you will make use of special products in factoring. In the latter part of the chapter you will use special products and factoring in solving equations.

[A]

REVIEW EXERCISES

Find the following products:

1. $8(a - b + c)$
2. $x^2(x^2 - x + 3)$
3. $3\,abc(a + b - c)$
4. $5\,xy(x^2 + y^2)$
5. $3\,a^n(a^n + a^{n-1})$
6. $-4\,x^3(x^2 + x - 1)$
7. $-5\,y^3(y^3 - 3\,y^2 - y)$
8. $ax^2(1 - 2\,x^2 - 3\,x^4)$
9. $6\,xy(\frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{6}y^2)$
10. $5\,x^a(x^{2a} - 3\,x^a + x^{a+1})$

**Product of Two Binomials** [A]

If two binomials have no like terms, their product is a polynomial of four terms.

For example, $(a + b)(c + d) = ac + bc + ad + bd$.

If two binomials have like terms, their product is either a binomial or a trinomial. The product at the right is a trinomial.

$$\begin{array}{r} 2\,x + 3\,y \\ x - 4\,y \\ \hline 2\,x^2 + 3\,xy \qquad\qquad \\ -8\,xy - 12\,y^2 \\ \hline 2\,x^2 - 5\,xy - 12\,y^2 \end{array}$$

RED ARROWS

Let us see how this product is found mentally. The $2\,x^2$ is found by multiplying $2\,x$ by $x$, the first terms of the binomials; the $-12\,y^2$ is found by multiplying $3\,y$ by $-4\,y$, the last terms of the binomials; and $-5\,xy$ is found by adding $3\,xy$ and $-8\,xy$, which are the cross products of the multiplication.

Example 1. Find the product $(3\,a + b)(2\,a - 5\,b)$.

**Solution.** The first term of the product is $(3\,a)(2\,a) = 6\,a^2$; the last term of the product is $(b)(-5\,b) = -5\,b^2$; one cross product is $2\,ab$, the other is $-15\,ab$; their sum is $-13\,ab$. Then

$(3\,a + b)(2\,a - 5\,b) = 6\,a^2 - 13\,ab - 5\,b^2$

$$(3\,\underbrace{a + \overbrace{b)(2}^{2\,ab}\,a - 5}_{-15\,ab}\,b)$$

RED TIE LINES

#6A

COLOR

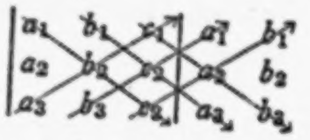
LINES USED TO ILLUSTRATE
OPERATIONS TO BE PERFORMED

### Determinants of the Third Order[A]

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

having 3 rows and 3 columns, is a determinant of the *third order*. It represents the expression $a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3$, which is called the expansion of the determinant. $a_1b_2c_3$ is the principal diagonal.

The expansion just given can be written by different methods. One method is as follows:


$a_1$ $b_1$ $c_1$ $a_1$ $b_1$
$a_2$ $b_2$ $c_2$ $a_2$ $b_2$
$a_3$ $b_3$ $c_3$ $a_3$ $b_3$

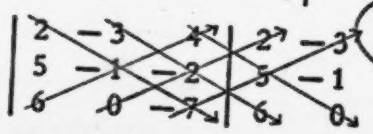
REP LINES

1. Repeat the first two columns of the determinant at the right of the determinant.
2. Draw arrows through the diagonals, as shown.
3. Find the product of each set of three elements through which an arrow is drawn, giving each product a plus sign if the arrow points down and a minus sign if the arrow points up.

This method of evaluating a third-order determinant does not apply to any other higher order.

**Example.** Evaluate $\begin{vmatrix} 2 & -3 & 4 \\ 5 & -1 & -2 \\ 6 & 0 & -7 \end{vmatrix}$

**Solution.**



REP LINES

$$= (2)(-1)(-7) + (-3)(-2)(6) + (4)(5)(0) - (6)(-1)(4) - (0)(-2)(2) - (-7)(5)(-3)$$

$$= 14 + 36 + 0 + 24 - 0 - 105 = -31$$

**EXERCISES**

Evaluate: [A]

1. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 5 & 6 \end{vmatrix}$ 2. $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ 3. $\begin{vmatrix} a & 0 & 1 \\ b & 1 & 0 \\ c & c & 1 \end{vmatrix}$

#6A

COLOR

LINES USED TO ILLUSTRATE
OPERATIONS TO BE PERFORMED

number in the other. In the equation $3n + 4 = 25$, we may proceed as follows:

**Solution.** $$3n + 4 = 25$$

STEP 1. If we subtract 4 from each member, we obtain
$$3n = 21$$

STEP 2. If we then divide each member by 3, we obtain
$$n = 7$$

RED BACKGROUND

Did you notice that in the words underlined at the foot of page 40 we proposed to do two things: (1) to undo the ties that hold the unknown to other numbers, and (2) to keep the two members equal at all times?

To accomplish the first of these tasks, that is, to undo the ties, we used the idea of inverse processes. We know that an addition will undo a subtraction, and vice versa, and we know that a multiplication will undo a division, and vice versa. In Step 1 of the solution we *subtracted 4* because, in the original equation, *4 had been added* to the term containing the unknown. In Step 2 of the solution we *divided by 3* because, in the equation $3n = 21$, the unknown was *multiplied by 3.*

To accomplish the second task, that is, to keep the two members of each succeeding equation equal, we made use of statements 2 and 4 of the following assumptions. These assumptions (sometimes called axioms) are statements which everyone believes true, even though we cannot prove them.

RED LINE

**Our Basic Assumptions about Equation Solving**

1. If the same number is added to each member of an equation, the members of the resulting equation are equal.
2. If the same number is subtracted from each member of an equation, the members of the resulting equation are equal.
3. If each member of an equation is multiplied by the same number, the members of the resulting equation are equal.
4. If each member of an equation is divided by the same number (except 0), the members of the resulting equation are equal.
5. A quantity may be substituted for its equal in any expression.

#7A

COLOR

BOX OR BACKGROUND USED
FOR EMPHASIS

## Graphical Representation [A]

In statistical work we frequently make use of graphs. A graph with vertical bars representing the frequencies of the various classes along the horizontal axis is called a histogram. In a histogram usually no space is left between the bars. The histogram at the right shows the scores from the table on page 468. Notice that the scores 70, 80, and so on are written a tiny bit to the right of the edge of a bar. That is because the classes are separated at the points 69.5, 79.5, and so on. The histogram shows that a score of 70 falls within the first group, a score of 80 within the second, and so on.

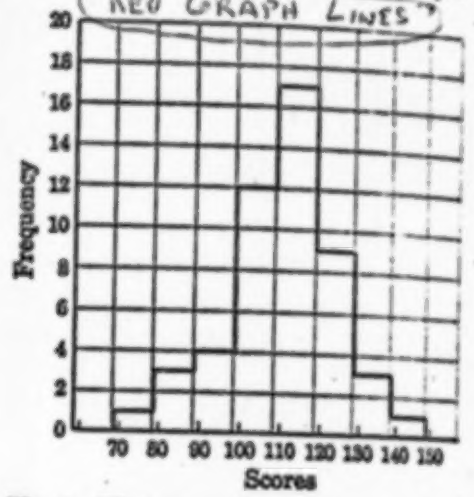


**Fig. 1. Histogram of scores made in a state algebra contest**

Sometimes a line graph called a frequency polygon is used. In a frequency polygon lines are drawn connecting the midpoints of the class intervals. A frequency polygon of the scores in the histogram above is shown in Fig. 2. The frequency polygon has been drawn on top of the histogram to show how it connects the midpoints of the class intervals.

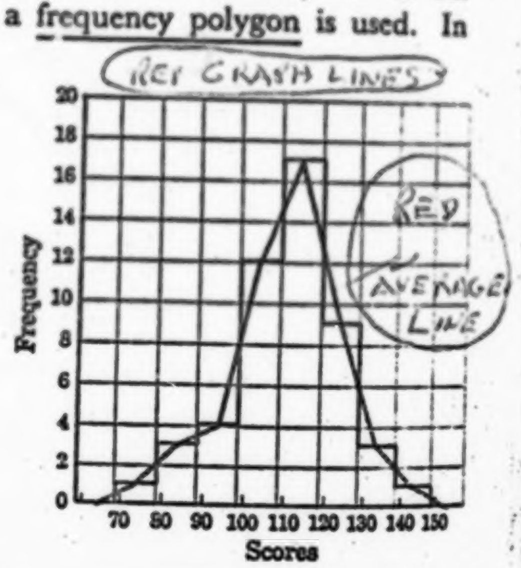


**Fig. 2. Frequency polygon of scores made on state algebra contest**

Notice that both the histogram and the frequency polygon show a slight bell-shape. Had the test been given to thousands of algebra pupils of similar ability instead of to fifty, the chances are that the scores would form an almost perfect bell-shaped curve, as shown in Fig. 3 on the next page. This curve is known as the normal probability curve.

#8A

COLOR

GRAPHS

3. The cost of an article equals the selling price decreased by the margin.

4. The selling price of an article is equal to its cost plus the profit plus the overhead.

5. To find the volume of a cylinder, multiply its altitude by the area of its base.

6. The volume of a pyramid equals one third the product of its base and altitude.

7. To find the amount, add the principal and interest.

8. To find the diameter of a circle, divide the circumference by $\pi$.

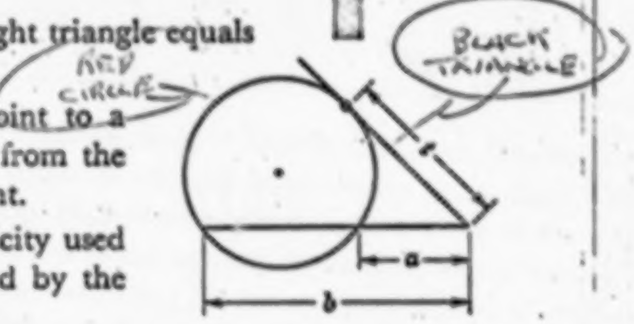
9. The sum of the angles $A$, $B$, and $C$ of a triangle is 180°.

10. To find the number of degrees in the sum of the angles of a polygon, subtract 2 from the number of its sides and multiply the difference by 180.

11. The average of four numbers is found by dividing their sum by 4.

12. The square of the hypotenuse of a right triangle equals the sum of the squares of the two legs.

13. The square of a tangent from a point to a circle is equal to the product of a secant from the point and the external segment of the secant.



14. The number of watt-hours of electricity used is equal to the number of watts multiplied by the number of hours.

15. To find the number of lines that are determined by a given number of points, no three of which are collinear, subtract 1 from the number of points, multiply the remainder by the number of points, and take half the product.

### Making Formulas from Data [A]

In the exercises above you made the formulas from given relationships. In the following exercises you are to determine the relationships and make the formulas.

[A] EXERCISES

Write a formula showing the relationship in each exercise:

1. The number of seats in a room having 8 seats in a row.

#9A

COLOR

GEOMETRIC FIGURES

Since the original scores are given accurate to the nearest unit we shall make the break between classes at the point half-way between two units, thus:

$$\underbrace{\cdots 78\quad 79}_{69.5-79.5}\overset{79.5}{,}\;\underbrace{80\quad 81\quad 82\quad 83 \cdots 87\quad 88\quad 89}_{79.5-89.5}\overset{89.5}{,}\;\underbrace{90\quad 91\quad 92 \cdots}_{89.5-99.5}$$

This means that the class units will be 69.5–79.5, 79.5–89.5, and so on. Our distribution will appear as follows:

FREQUENCY DISTRIBUTION FOR ALGEBRA CONTEST SCORES

RED LINES

| Class | m | f | fm | Cumulative Frequency |
|---|---|---|---|---|
| 139.5–149.5 | 144.5 | 1 | 144.5 | 1 |
| 129.5–139.5 | 134.5 | 3 | 403.5 | 4 |
| 119.5–129.5 | 124.5 | 9 | 1120.5 | 13 |
| 109.5–119.5 | 114.5 | 17 | 1946.5 | 30 |
| 99.5–109.5 | 104.5 | 12 | 1254.0 | 42 |
| 89.5–99.5 | 94.5 | 4 | 378.0 | 46 |
| 79.5–89.5 | 84.5 | 3 | 253.5 | 49 |
| 69.5–79.5 | 74.5 | 1 | 74.5 | 50 |
| | | | $\Sigma fm = 5575.0$ | |

When we arrange data in classes we must choose one number to *represent* the value of the numbers within each class. We have used the midpoint of each class as the representative of the class, and have written the values in the column headed *m*, which stands for *midpoint*. The frequency column (*f*) shows the frequency within each class, and the *fm* column shows the *product* of the frequency and the class value. The *fm* column makes it easy to find the arithmetic mean of the class values. Do you see that if we divide 5575.0, the sum of the terms in the *fm* column, by 50, we get the arithmetic mean of the class values—111.5? In the table, $\Sigma fm$ means "the sum of the *fm* terms." The symbol $\Sigma$ (pronounced sigma) is a Greek letter often used to denote "the sum of."

For scores arranged by classes we use the arithmetic mean of the class values as the arithmetic mean of the scores. Obviously, however, this arithmetic mean is not identical with the arithmetic mean of the actual scores because when scores are grouped, the individual scores lose their identity and are represented by their class values. The arithmetic mean of the actual scores above is 111.3.

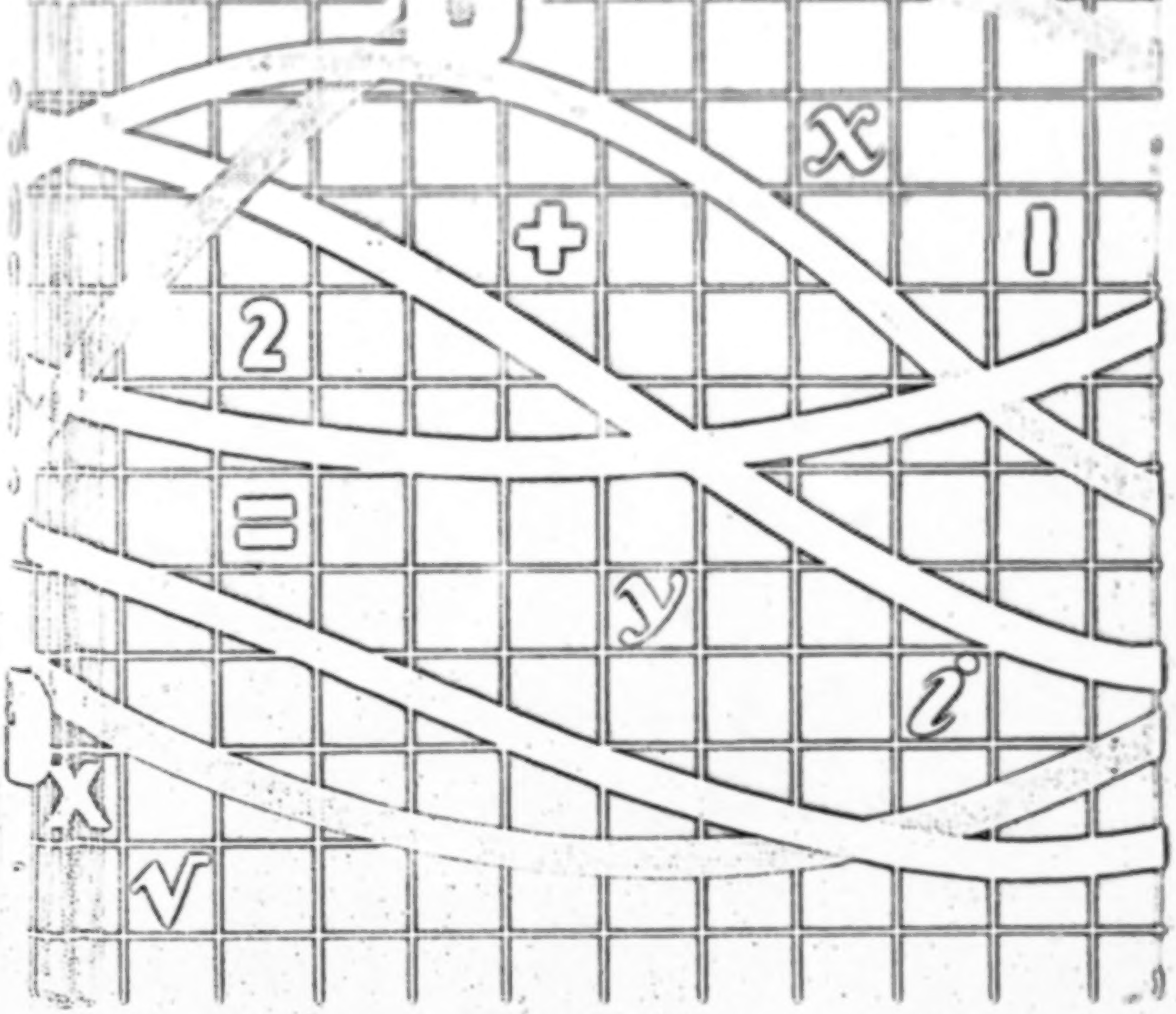
#10A

COLOR

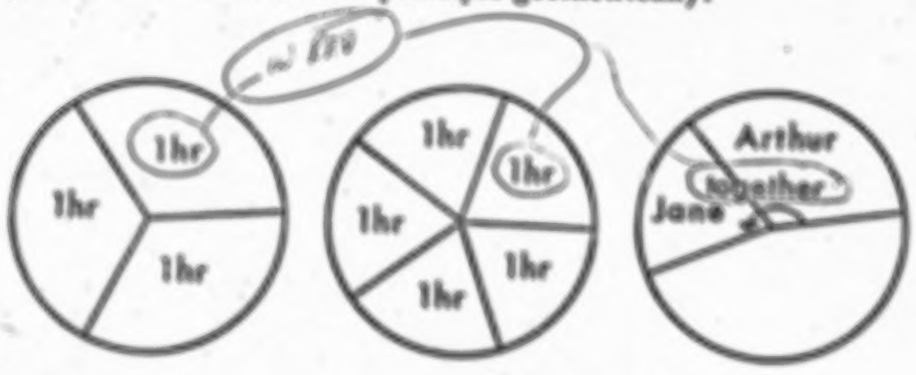
TABLES

SAMPLES FROM

# INTERMEDIATE
# Algebra



We solve problems of this sort on the basis of the amount of work done in one hour. For example, in the above problem Arthur takes 3 hours to do his job, so on the average he will do $\frac{1}{3}$ *of it in one hour*, $\frac{2}{3}$ of it in two hours. Jane would do $\frac{1}{5}$ *of the job each hour*. If they work together, they will do $\frac{1}{3}+\frac{1}{5}$ of the job in one hour. The figures below illustrate this principle geometrically:



Arthur
$\frac{1}{3}$ of the job
each hour

Jane
$\frac{1}{5}$ of the job
each hour

Together
$\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$
of the job
each hour

Now, suppose we let $x =$ the number of hours required working together. Arthur does $\frac{1}{3}$ of the job each hour and works $x$ hours. *So he does $x$ times $\frac{1}{3}$ or $\frac{x}{3}$ of the entire job. Why?*

Jane does $\frac{1}{5}$ of the job each hour and works $x$ hours. She does $x$ times $\frac{1}{5}$ or $\frac{x}{5}$ of the entire job. Why?

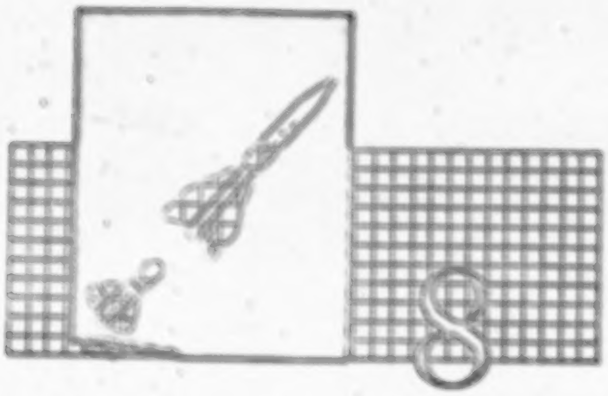
So we have the equation $\frac{x}{3}+\frac{x}{5}=1$.

When two people work on a job together, each does a fractional part of it, the sum of the fractions being the complete job, or 1. For example, if one person does $\frac{3}{5}$ of a job and another does $\frac{2}{5}$, the job is done. $\frac{3}{5}+\frac{2}{5}=1$.

Clearing our equation of fractions we have

$$5x+3x=15$$
$$x=1\tfrac{7}{8}\text{ hours}$$

#2A



## 8

# EXPONENTS

By this time you have become quite familiar with exponents that are positive integers, and with the laws governing their use. But any number may be an exponent, and in this chapter you will learn the meanings of certain new kinds of exponents, and how to use them.

## 85 Laws of Exponents

We summarize here the laws of exponents as you have known them.

★ **Laws**

**I.** For multiplication when the bases are alike:

$$a^m \cdot a^n = a^{m+n}$$

Illustration $x^3 \cdot x^5 = x^8$

**II.** For multiplication when the bases are different, but the exponents alike:

$$a^n \cdot b^n = (ab)^n$$

Illustration $2^3 \cdot 5^3 = 10^3$

**III.** For division when the bases are alike:

$$\frac{a^m}{a^n} = a^{m-n}$$

Illustration $\frac{x^5}{x^3} = x^2$

REV

✓

#5.

COLOR

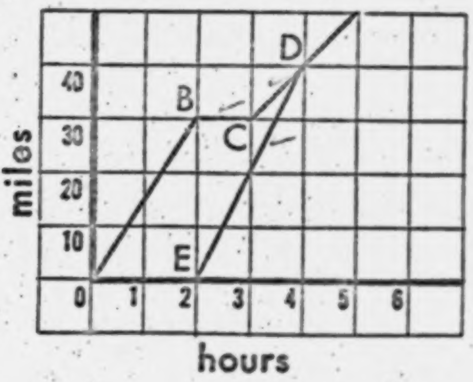
USED TO EMPHASIZE A WORD
OR NUMBER

have the straight line *OB*. In like manner a rate of 5 miles an hour is represented by the line *OC*. These lines do not represent the path of the traveler, but the slope of the lines represents the rate. The steeper the line the greater the rate represented.

### ★ Example I

John leaves a certain point on his bicycle at the rate of 15 miles an hour. Two hours later he has an accident which delays him 1 hour, after which he proceeds at 10 miles an hour. Two hours after John started, James started to overtake him, riding on his bicycle at the rate of 20 miles an hour. How long and how far must he ride?

*Solution:* Draw *OB* to represent the rate of John and continue it until it reaches the ordinate corresponding to 2 hours. He was then delayed an hour, hence time went on, but distance did not change. The line *BC* becomes, therefore, horizontal at this point. Then *CD* is drawn in a direction to indicate 10 miles an hour, and continued indefinitely.



RED #3 LINES

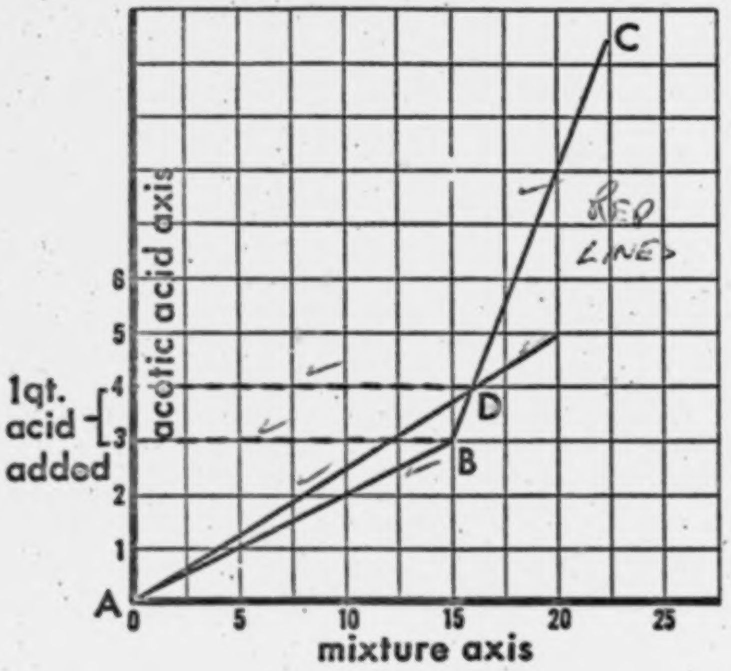
The fact that James started two hours later than John is indicated by starting the line *ED* two units to the right of the origin, and the slope of the line *ED* represents a rate of 20 miles an hour. The intersection *D* shows that John's rate and James's rate have caused them to reach a distance of 40 miles simultaneously. Hence James has overtaken John. From the axes we read a distance of 40 miles and a time of 2 hours.

#8A

we draw a line $AD$ at such a slope as to represent 1 quart of alcohol in 6 quarts of the mixture. This line meets the horizontal line at $O$. Then $BD$ represents the amount of water added. The graph indicates that this represents 10 quarts, which agrees with the conditions of the problem.

**★ Example V**

How many quarts of pure acetic acid must be added to 15 quarts of a 20% solution to produce a 25% solution?



*Solution:* Lay off units representing quarts of the mixture on the horizontal axis, and quarts of acetic acid on the vertical axis. The line $AB$ is drawn so as to represent 3 quarts of the acetic acid in 15 quarts of the mixture. $BC$ is drawn to represent the acetic acid added. The angle at which it is drawn shows that when one quart is added to the acetic acid, a quart is also added to the mixture. The line $AD$ represents the resulting 25% solution. It shows 1 quart of the acid in 4 quarts of the mixture.. The amount added is 1 quart.

---

#8A COLOR

GRAPHS

27. Find to the nearest tenth the number of rods in the side of a square field which contains 8 acres. *Hint:* 1 acre = 160 square rods.

28. The area of a triangle whose sides are represented by $a$, $b$, and $c$ is $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$.

Find the area of a triangle whose sides are 24, 18, and 16 inches, respectively.

## Exercises

**Group A**

In the right triangle the square of the longest side equals the sum of the squares of the other two sides.

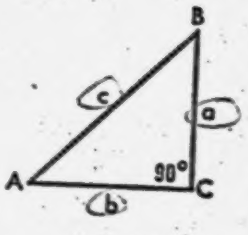
Thus in the adjoining figure

$$a^2 + b^2 = c^2$$

Hence $c = \sqrt{a^2 + b^2}$

Also $a = \sqrt{c^2 - b^2}$

and $b = \sqrt{c^2 - a^2}$



REV
LOWER
CASE

1. Find the value of $c$ in a right triangle whose altitude $(a) = 8$ and whose base $(b) = 6$.

2. Solve the equation $c^2 = a^2 + b^2$ for $a^2$. For $a$. Then find the value of $a$ if $c = 5$ and $b = 4$.

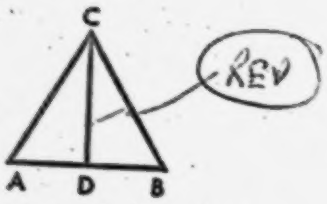
3. Solve the equation $c^2 = a^2 + b^2$ for $b^2$. For $b$. Then find the value of $b$ if $c = 5$ and $a = 3$.

4. A ladder 25 feet long is even with the top of a wall $a$ feet high. The foot of the ladder is 13.5 feet away from the foot of the wall. How high is the wall?

5. A railway incline is 5 in each 150 (*i.e.* 5 feet high for each 150 feet of slant height, or when $c = 150$, $a = 5$). Find the projected or horizontal length.

6. A perpendicular is drawn from the vertex to the base of an equilateral (all sides equal) triangle each of whose sides is 4 inches.

Find the length of the perpendicular $CD$.



7. The side of a square is 2 inches. Find the length of the straight line which joins two opposite corners of the square.

#9A

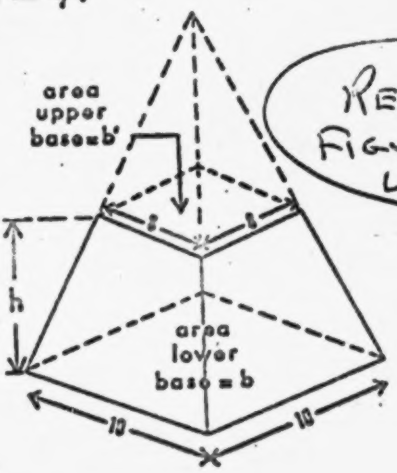
COLOR

GEOMETRIC FIGURES

14. $C_0 = 2\sqrt{\pi A}$, where $C_0 =$ the circumference of a circle and $A =$ area of the circle. Find $C_0$ if $A = 36$.

15. (a) Solve $\pi d^2 = 4A$ for $d$.
(b) Find $d$ when $A = \frac{176}{7}$ and $\pi = \frac{22}{7}$.

16. The volume of a frustum of a pyramid is expressed by the formula $V = \frac{1}{3}h(b + b' + \sqrt{bb'})$, where $h =$ the altitude and $b$ and $b'$ are the bases. If $h = 3$ feet and the bases are squares of sides 10 inches and 8 inches respectively, compute the volume to the nearest tenth of a cubic inch.



RED FIGURE LINES

17. The horse power of a certain gasoline engine is denoted by $H = \frac{1}{6}D^2N$, where $D$ equals the diameter of a cylinder in inches and $N$ equals the number of cylinders. Solve this formula for $D$. Compute the diameter of the cylinder of a 20 horse-power engine of 6 cylinders.

18. The time in seconds ($t$) for the complete vibration of a simple pendulum is given by the formula $t = \pi\sqrt{\frac{l}{32.2}}$, where $l$ represents the length in feet of the pendulum and $\pi = 3.1416$. Find $l$ if a pendulum vibrates in 2 seconds.

## Exercises

### Group A

1. Find the side of a square whose area is 27 square inches.

2. What is the radius of a circle whose area is 24.2 square feet?

3. What is the diagonal of a square whose side is 6?

4. What is the diagonal ($d$) of a square whose side is $s$?

5. By means of the equation obtained in number 4, what is the diagonal of a square whose side is 8? 10? 15? 20?

6. Solve your answer obtained in exercise 4 for the value of $s$.

7. What is the side of a square whose diagonal is $\sqrt{50}$?

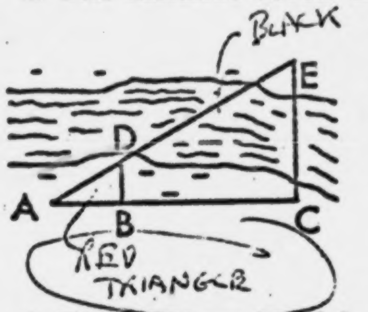
#9A.

COLOR

GEOMETRIC FIGURES

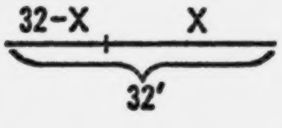
**14.** If a man can dig a trench 15 feet long in $3\frac{1}{2}$ hours, how long a trench would you expect him to be able to dig in $6\frac{1}{3}$ hours at the same rate?

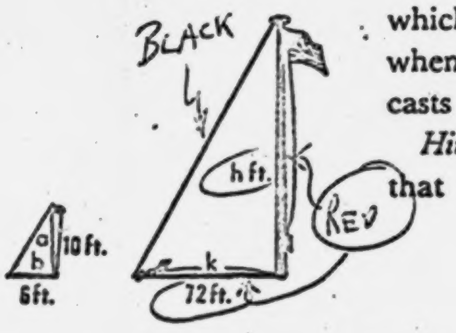
**Group B**



**1.** The distance across a river or an impassable barrier may be found by the use of proportion as follows. The distance *AB* is to *AC* as *BD* is to *CE*. If *AB* = 10 inches, *BD* = 5 inches, and *AC* measures 800 feet, what is the width *CE*? Note that *ABD* and *ACE* are right triangles.

**2.** Find the lengths of the segments of a line 32 feet long, if *x*, one part, is a mean proportional between the whole line and the other part.



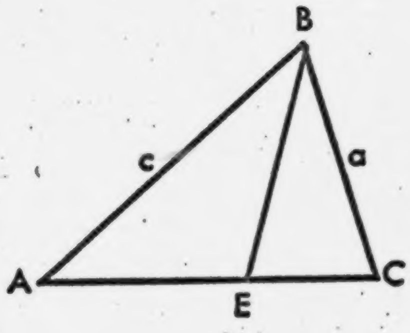


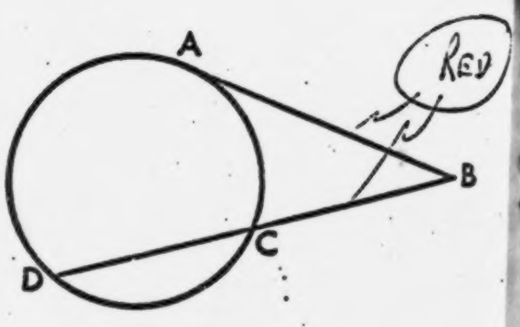
**3.** Find the height of a flagstaff which throws a shadow of 72 feet, when a vertical rod 10 feet long casts a shadow of 6 feet.

*Hint:* From geometry we know that

$$\frac{b}{k}=\frac{a}{h} \text{ or } \frac{6}{72}=\frac{10}{h}$$

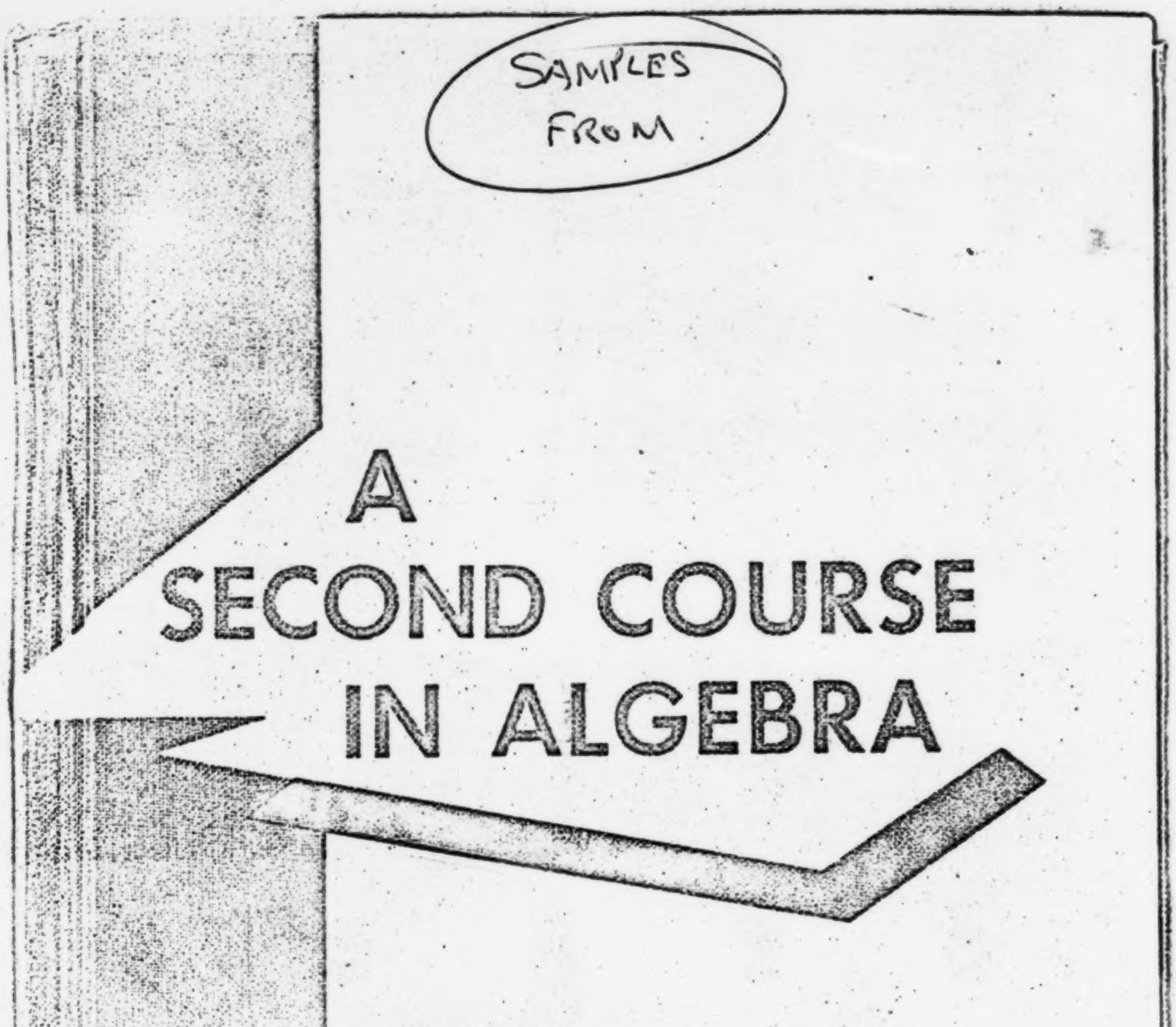
**4.** In the triangle *ABC* below, *BE* bisects angle *ABC*. Then, by geometry, *AE* : *EC* = *c* : *a*. If *c* = 9, *a* = 6 and *AC* = 10, find *AE* and *EC*.





#9A

SAMPLES FROM



# A SECOND COURSE IN ALGEBRA

LENNES
MAUCKER
KINSELLA

## AN EXAMPLE IN SOLVING EQUATIONS

Let us study the solution shown below.

Step 1. In Equation (1) multiply $x + 2$ by 3; that is, for $3(x + 2)$ write $3x + 6$. This gives Equation (2).
Step 2. Subtract 6 from both members. This gives Equation (3).
Step 3. Subtract $3x$ from both members. This gives Equation (4).
Step 4. Divide both members by 2. This gives $x = 4$.

$$\begin{aligned} 2x + 3(x + 2) &= 3x + 14 && (1)\\ 2x + 3x + 6 &= 3x + 14 && (2)\\ 2x + 3x &= 3x + 8 && (3)\\ 2x &= 8 && (4)\\ x &= 4 \end{aligned}$$

To check, substitute 4 for $x$ in both members of the original equation. As shown below, this reduces both members to 26; that is, $x = 4$ satisfies the given equation.

Check

$$2 \cdot 4 + 3(4 + 2) = 26$$

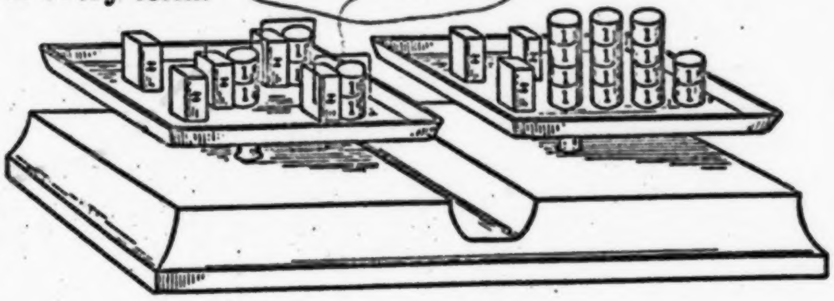
Also:

$$3 \cdot 4 + 14 = 26$$

In practice we would probably pass directly from (2) to (4) by subtracting at once both 6 and $3x$, omitting Equation (3).

If we admit that we may use any of the five operations enumerated on page 61, what is to guide us in selecting the proper ones to use? The answer is that we are working toward a certain end—toward arriving at an equation in which the unknown and nothing else will be in one member, and only in one member. We must approach each equation individually, use our intelligence, and combine these operations so as to reach such an end. Note that multiplying both members of an equation by − 1 changes the sign of every term.

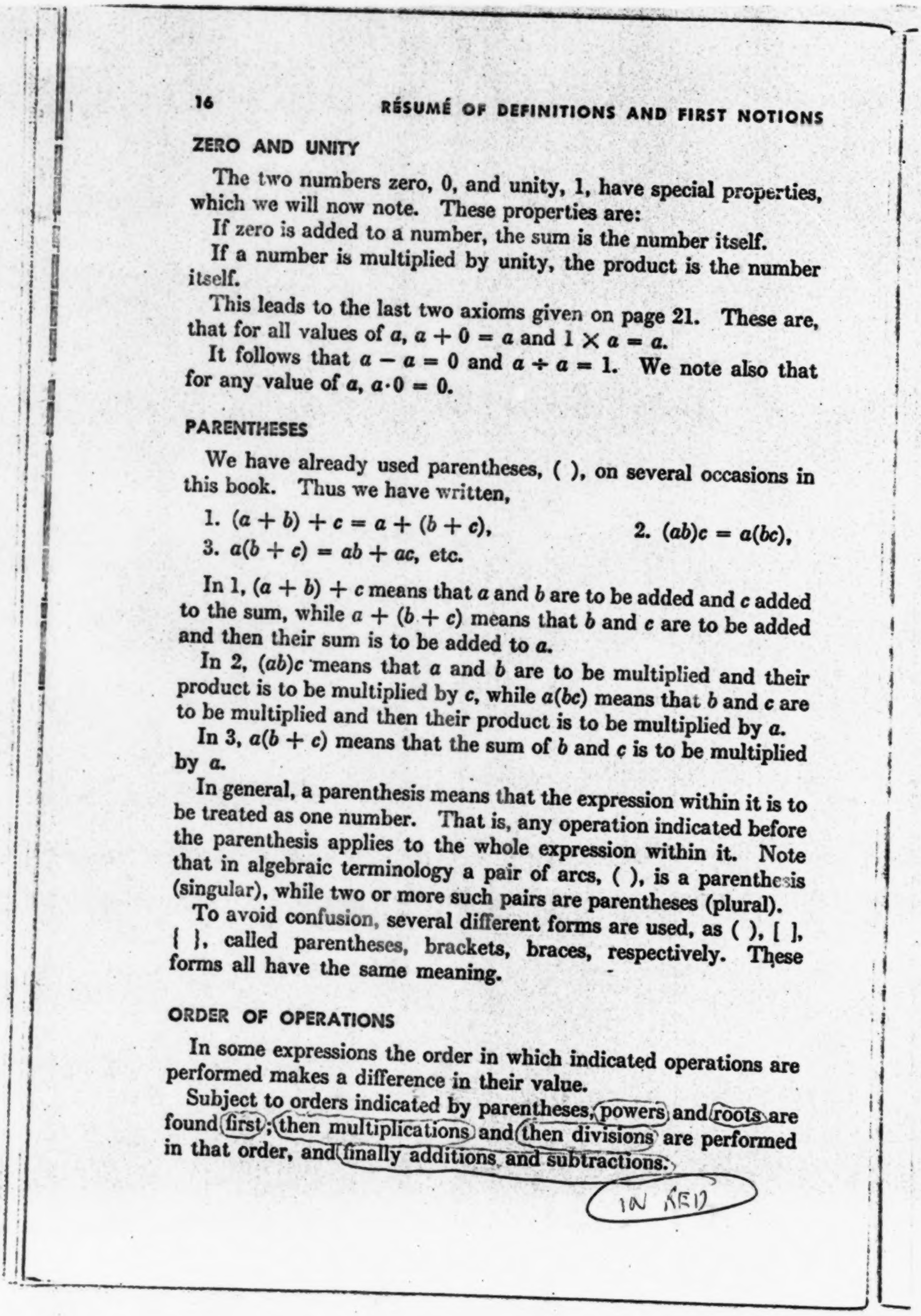
RED BRACKETS



#2A.

COLOR

LINE DRAWINGS AND DIAGRAMS



### ZERO AND UNITY

The two numbers zero, 0, and unity, 1, have special properties, which we will now note. These properties are:

If zero is added to a number, the sum is the number itself.

If a number is multiplied by unity, the product is the number itself.

This leads to the last two axioms given on page 21. These are, that for all values of $a$, $a + 0 = a$ and $1 \times a = a$.

It follows that $a - a = 0$ and $a \div a = 1$. We note also that for any value of $a$, $a \cdot 0 = 0$.

### PARENTHESES

We have already used parentheses, ( ), on several occasions in this book. Thus we have written,

1. $(a + b) + c = a + (b + c)$,
2. $(ab)c = a(bc)$,
3. $a(b + c) = ab + ac$, etc.

In 1, $(a + b) + c$ means that $a$ and $b$ are to be added and $c$ added to the sum, while $a + (b + c)$ means that $b$ and $c$ are to be added and then their sum is to be added to $a$.

In 2, $(ab)c$ means that $a$ and $b$ are to be multiplied and their product is to be multiplied by $c$, while $a(bc)$ means that $b$ and $c$ are to be multiplied and then their product is to be multiplied by $a$.

In 3, $a(b + c)$ means that the sum of $b$ and $c$ is to be multiplied by $a$.

In general, a parenthesis means that the expression within it is to be treated as one number. That is, any operation indicated before the parenthesis applies to the whole expression within it. Note that in algebraic terminology a pair of arcs, ( ), is a parenthesis (singular), while two or more such pairs are parentheses (plural).

To avoid confusion, several different forms are used, as ( ), [ ], { }, called parentheses, brackets, braces, respectively. These forms all have the same meaning.

### ORDER OF OPERATIONS

In some expressions the order in which indicated operations are performed makes a difference in their value.

Subject to orders indicated by parentheses, powers and roots are found first; then multiplications and then divisions are performed in that order, and finally additions and subtractions.

IN RED

#5

COLOR

USED TO EMPHASIZE A WORD OR NUMBER

**Products of the type $(ax + b)(cx + d)$.** In the multiplication of two binomials such as those shown in the example at left below, the products $2x \cdot 3x = 6x^2$ and $3 \cdot 4 = 12$ are called **end-products.** The products $3x \cdot 3 = 9x$ and $2x \cdot 4 = 8x$ are called **cross-products.** In the final product, the term $17x$ is the sum of the cross-products.

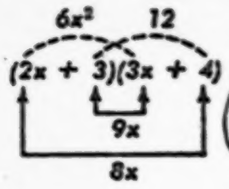
$$\begin{array}{r} 2x + 3 \\ \underline{3x + 4} \\ 6x^2 + 9x \phantom{+ 12} \\ \underline{8x + 12} \\ 6x^2 + 17x + 12 \end{array} \qquad \begin{array}{r} 3x - 1 \\ \underline{4x + 7} \\ 12x^2 - 4x \phantom{- 7} \\ \underline{21x - 7} \\ 12x^2 + 17x - 7 \end{array}$$

In the example at right above, what are the end-products? What are the cross-products? What is their sum?

Such products can be read at sight. The scheme at the right will help.

The end-products are shown by the dotted lines above and the cross-products are shown by the solid lines below.

The rule is:



RED LINES

*The product of two binomials is the sum of the end-products plus the sum of the cross-products.*

The sum of the cross-products is written in the middle.

Note that the rule applies word for word, whether the terms in the binomials are positive or negative.

## SIGHT WORK (A)

Read the products in the following.

1. $(2x + 3)(x + 1)$
2. $(x + 4)(3x + 2)$
3. $(2x + 5)(2x - 1)$
4. $(2x - 3)(2x + 5)$
5. $(5x + 1)(x - 9)$
6. $(7x + 2)(2x - 3)$
7. $(3x + 4)(x - 9)$
8. $(4x - 4)(x + 2)$
9. $(2x - 5)(3x - 1)$
10. $(5m + 2)(3m + 4)$
11. $(7m + 3)(5m - 2)$
12. $(8a + 3)(2a - 7)$
13. $(2y - 6)(y - 2)$
14. $(3a - 8)(7a - 2)$
15. $(4k - 3)(2k - 5)$
16. $(5c - 2)(4c - 1)$
17. $(-2x + 3)(x - 1)$
18. $(-2x - 1)(-3x - 2)$

**Factoring trinomials of the form $ax^2 + bx + c$.** An expression of the form $ax^2 + bx + c$ is called a quadratic trinomial.

6A

COLOR

LINES USED TO ILLUSTRATE
OPERATIONS TO BE PERFORMED

### RULES AND FORMULAS

In your study of arithmetic you learned the two rules given at the left below, and in your study of *A First Course in Algebra* you learned to translate these rules into the formulas given at the right.

RED BACKGRND

| *Arithmetic* | *Algebra* |
|---|---|
| *The area of a rectangle equals its length multiplied by its width.* | $A = wl$ |
| *The product of two fractions is a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators.* | $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ |

The formulas are complete statements of the rules. We say that the rules have been translated into formulas.

In the formulas the letters $w$, $l$, $a$, $b$, $c$, and $d$ represent "any numbers." The idea of "any number" is contained also in the rules, since they apply to all rectangles or fractions. Therefore, the idea of "any number" or "general number" is also present in arithmetic. In algebra we use letters to represent "general numbers."

### THE MAIN POINTS OF DIFFERENCE BETWEEN ARITHMETIC AND ALGEBRA

Algebra differs from arithmetic mainly in the following ways.

1. In algebra, letters are used to represent numbers.
2. In algebra, the equation is developed and used to solve problems.
3. The number system of arithmetic is enlarged by the introduction of signed numbers and of complex numbers.

### ALGEBRAIC SYMBOLS

The most important kinds of new symbols in algebra are the letters which stand for numbers. Numbers themselves are, of course, symbols, but we use them in algebra the same way that we use them in arithmetic. When symbols such as $+$, $-$, $\times$, $\div$, $=$, and $\sqrt{\phantom{x}}$ are used in algebra, they have the same meaning that they have in arithmetic. The raised dot is sometimes used to indicate multiplication. But where letters are involved, we may omit the multiplication sign. Thus $2 \times a \times b = 2 \cdot a \cdot b = 2ab$. We also indicate multiplication in algebra by placing our terms in parentheses and putting these next to each other, $(2)(a)(b)$.

#7A -

COLOR

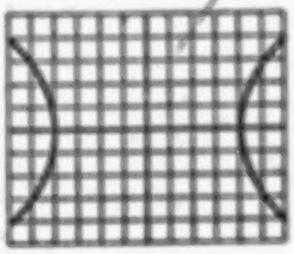
BOX OR BACKGROUND USED
FOR EMPHASIS

### EXERCISE (C)

Solve simultaneously Equations (1) and (2) above, and also (1) and (4). Solve simultaneously (1) and $x^2 + y^2 = 4$, and also (1) and $x^2 + y^2 = 36$. Interpret each of these solutions.

## THE HYPERBOLA AND THE CIRCLE

REV GRAPH GRD



We shall now study the equation

$$x^2 - y^2 = 16. \qquad (1)$$

Verify that this equation is satisfied by

$$\begin{cases} x = 4 \\ y = 0 \end{cases} \qquad \begin{cases} x = -4 \\ y = 0 \end{cases} \qquad \begin{cases} x = 5 \\ y = 3 \end{cases}$$

$$\begin{cases} x = 5 \\ y = -3 \end{cases} \qquad \begin{cases} x = -5 \\ y = 3 \end{cases} \qquad \begin{cases} x = -5 \\ y = -3 \end{cases}$$

From the form $y^2 = x^2 - 16$ verify that:

a. The graph of Equation (1) has no point for which $x$ is between $-4$ and $4$.

b. As $x$ grows large beyond 4 in the positive direction, $y$ grows large both in the positive and negative directions.

c. As $x$ grows large in absolute value, but in the negative direction, $y$ grows as in b.

The graph consists of the two branches shown in the figure. This curve is called the hyperbola.

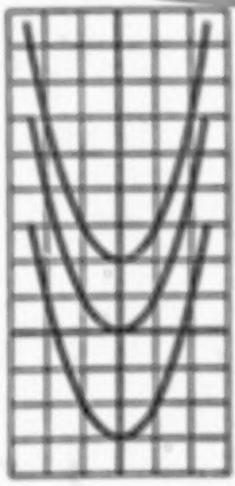
### EXERCISES (C)

Solve the following pairs of equations, and interpret the solutions. Construct the figures.

1. $\begin{cases} x^2 - y^2 = 16 \\ x^2 + y^2 = 25 \end{cases}$
2. $\begin{cases} x^2 - y^2 = 16 \\ x^2 + y^2 = 16 \end{cases}$
3. $\begin{cases} x^2 - y^2 = 16 \\ x^2 + y^2 = 9 \end{cases}$

## THE PARABOLA AND THE CIRCLE

REV GRID



We have already studied the parabola $y = x^2$ (see page 285). The equation $y = x^2 + 2$ represents the same curve raised two units.

Similarly, $y = x^2 - 3$ represents the same curve lowered three units.

The parabolas are shown in the figure.

#8A

COLOR

GRAPHS

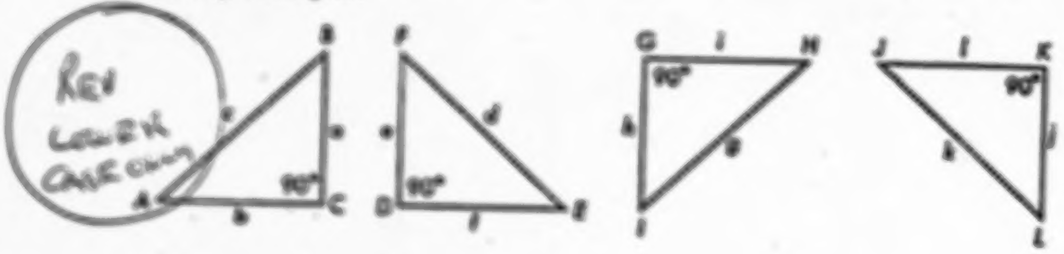
In the case of angle $B$, the other acute angle, the side opposite is not $a$ but $b$; the side adjacent is $a$ while the hypotenuse is still $c$. Hence,

$$\sin B = \frac{b}{c}, \qquad \cos B = \frac{a}{c}, \qquad \tan B = \frac{b}{a}.$$

Notice that the $\sin A = \cos B$, $\cos A = \sin B$, and $\tan A = \dfrac{1}{\tan B}$. We do not write any ratios for the right angle.

## SIGHT EXERCISES (A)

State the sine, cosine, and tangent ratios for each acute angle in the following triangles.

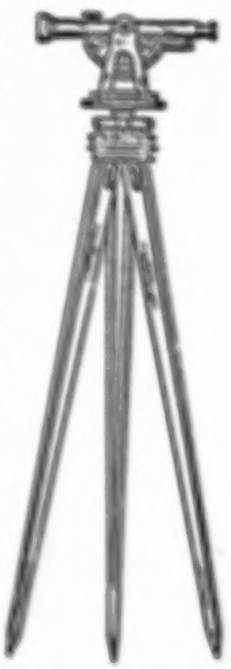


Key
Lower
Case [illegible]

### MEASURING ANGLES

You have already learned that a right angle is divided into 90 equal parts, each part being called a degree, written 1°. Seven degrees is written 7°, 58 degrees is written 58°, while 16 degrees and 30 minutes is written 16° 30′, and so on.

In practical surveying an instrument called a transit, shown at the right, is used to measure angles.



For less accurate surveying, often called reconnaissance surveying, a compass is used. With the transit, angles can be measured to the nearest minute, while with the compass it is difficult to measure an angle closer than 10 minutes. In surveying, especially over rough ground, it is difficult to measure directly the straight-line distance between two points. You can easily see that it would be very difficult to measure accurately the distance across a lake or river,

#9A COLOR

GEOMETRIC FIGURES

use 10 as the base of a system of logarithms. Logarithms to the base 10 are called **common logarithms.** These are the logarithms we shall use in our computations in this chapter.

The most important equation in this chapter is

$$10^x = N.$$

If $x$ is any positive number, you can see that $N$ will be positive. If $x$ is a negative number, say $-3$, then $10^{-3} = \frac{1}{10^3} = .001$, which is also a positive number. In fact, if $x$ is any negative number, $10^x = N$ will be positive. Since we say that logarithm of $N$ to the base 10 is $x$, we do not have the logarithms of negative numbers. $N$ will always be a positive number. On the other hand, the logarithms of some numbers will be negative numbers, even though the numbers themselves are not negative. For example, as we showed above, the logarithm of .001 is $-3$, although .001 is a positive number.

## APPROXIMATE LOGARITHMS; CHARACTERISTIC AND MANTISSA

There are only a few numbers whose logarithms can be given exactly as decimals. The following table illustrates some of them.

| *Exponent Form* | *Logarithm Form* |
|---|---|
| $10^3 = 1{,}000$ | $\log 1000 = 3$ |
| $10^2 = 100$ | $\log 100 = 2$ |
| $10^1 = 10$ | $\log 10 = 1$ |
| $10^0 = 1$ | $\log 1 = 0$ |
| $10^{-1} = .1$ | $\log .1 = -1$ |
| $10^{-2} = .01$ | $\log .01 = -2$ |
| $10^{-3} = .001$ | $\log .001 = -3$ |

RED EXPONENTS

REV

REV

In the right-hand part of the table we have used *log* as an abbreviation for *logarithm* and have omitted the base 10, since it is understood.

Suppose we have to obtain log 316.2. It turns out that $\sqrt{100{,}000}$ is approximately equal to 316.2. Hence, $(10^5)^{\frac{1}{2}} = 10^{2\frac{1}{2}} = 10^{2.5000} = 316.2$. Therefore, $\log 316.2 = 2.5000$, approximately. Since the cube root of 10,000 is approximately equal to 21.54, $(10^4)^{\frac{1}{3}} = 10^{1.3333} = 21.54$. Therefore, $\log 21.54 = 1.3333$. The integer 2 in 2.5000 and the integer 1 in 1.3333 are called the characteristics

#10A

COLOR

TABLES

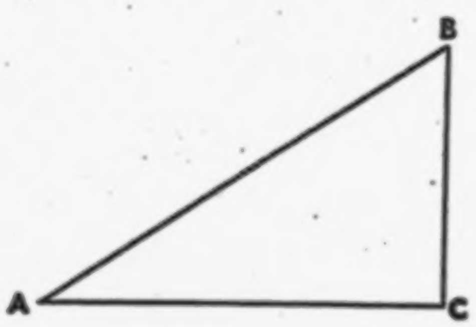
REV FIGURES

REV LINE

| Angle | Sin | Cos | Tan |
|---|---|---|---|
| 28° | .4695 | .8829 | .5317 |
| 29° | .4848 | .8746 | .5543 |
| 30° | .5000 | .8660 | .5774 |
| 31° | .5150 | .8572 | .6009 |
| 32° | .5299 | .8480 | .6249 |
| 33° | .5446 | .8387 | .6494 |
| 34° | .5592 | .8290 | .6745 |
| 35° | .5736 | .8192 | .7002 |
| 36° | .5878 | .8090 | .7265 |
| 37° | .6018 | .7986 | .7536 |
| 38° | .6157 | .7880 | .7813 |
| 39° | .6293 | .7771 | .8098 |
| 40° | .6428 | .7660 | .8391 |
| 41° | .6561 | .7547 | .8693 |
| 42° | .6691 | .7431 | .9004 |
| 43° | .6820 | .7314 | .9325 |
| 44° | .6947 | .7193 | .9657 |
| 45° | .7071 | .7071 | 1.0000 |

**Examples**

**1. In the right triangle *ABC*, angle $A = 34°$ and $AB = 48.26$. Find *AC* and *CB*.**



**To find *AC*, the side adjacent to angle *A*, we know *AB*, the hypotenuse. Hence, $\cos 34° = \frac{AC}{48.26}$. In the table, $\cos 34° = .8290$. Therefore, $AC = 48.26(.8290) = 40.01$, correct to four significant digits.**

**To find *CB*, the side opposite angle *A*, we know *AB*, the hypotenuse. Hence, $\sin 34° = .5592 = \frac{CB}{48.26}$, and $CB = 26.99$.**

10 Pt

COLOR

TABLES

# APPENDIX D
(Second Half)
# BIOLOGY

## American Standard

# Picture Sizes for Roll and 35mm Still-Film Cameras

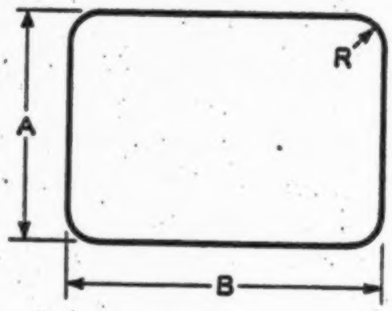
ASA
*Reg. U. S. Pat. Off.*
PH3.39-1961
Revision of
Z38.4.8-1950

*UDC 771.531.3

## 1. Scope

This standard applies to all roll-film still cameras using duplex paper numbering for metering with films conforming to American Standard Dimensions for Amateur Roll Film, Backing Paper, and Film Spools, PH1.21-1956 and to still cameras using 35mm (millimeter) film conforming to American Standard Dimensions for 35mm Film Magazines and Film for Still Picture Cameras, PH1.14-1959.

## 2. Picture Sizes



**For Cameras Using Roll Film**

| Nominal Picture Size Inches | A Width Inches | A Width mm | B Length Inches | B Length mm | R Maximum Corner Radius Inch | R Maximum Corner Radius mm |
|---|---|---|---|---|---|---|
| 1⅝ x 1⅝ | 1.625 | 41.3 | 1.625 | 41.3 | 0.016 | 0.4 |
| 1⅝ x 2¼ | 1.625 | 41.3 | 2.250 | 57.2 | 0.016 | 0.4 |
| 1⅝ x 2½ | 1.625 | 41.3 | 2.500 | 63.5 | 0.031 | 0.8 |
| 2¼ x 2¼ | 2.250 | 57.2 | 2.250 | 57.2 | 0.031 | 0.8 |
| 2¼ x 3¼ | 2.250 | 57.2 | 3.250 | 82.6 | 0.031 | 0.8 |

**For Cameras Using 35mm Film**

| Nominal Picture Size mm | A Width Inch | A Width mm | B Length Inches | B Length mm | R Maximum Corner Radius Inch | R Maximum Corner Radius mm |
|---|---|---|---|---|---|---|
| 18 x 24 | 0.709 | 18.00 | 0.964 | 24.50 | 0.016 | 0.4 |
| 24 x 24 | 0.964 | 24.50 | 0.964 | 24.50 | 0.016 | 0.4 |
| 24 x 36 | 0.964 | 24.50 | 1.429 | 36.30 | 0.016 | 0.4 |

## 3. Tolerances

Tolerances on dimensions A and B are ± 0.016 inch or ± 0.4mm for cameras using amateur roll film, and ± 0.012 inch or ± 0.3mm for cameras using 35mm film.

Approved August 14, 1961, by the American Standards Association, Incorporated
Sponsor: Photographic Standards Board

*Universal Decimal Classification

Copyright 1962 by American Standards Association, Incorporated
10 East 40th Street, New York 16, N. Y.

Printed in U. S. A.
ASA¾M262/40

# APPENDIX F

## JOINT STATEMENT ON SELF-INSTRUCTIONAL MATERIALS AND DEVICES

The following is a joint statement prepared by committees of the American Psychological Association, the American Educational Research Association, and the Department of Audio Visual Instruction of the National Education Association. The statement has been approved by the governing boards of the association represented.

## SELF-INSTRUCTIONAL MATERIALS AND DEVICES

The use of self-instructional programmed learning materials in teaching machines and similar devices represents a potential contribution of great importance to American education. But this contribution can be realized best only if users have information with which to evaluate self-instructional materials. Accordingly, the following interim guidelines have been prepared.

1. Teaching machines do not in themselves teach. Rather, the teaching is done by program of instructional materials presented by the teaching machine. Any evaluation of a teaching machine thus requires an assessment of the availability and quality of programs for each type of machine, as well as its mechanical dependability.

2. A variety of programmed materials is becoming available, but not all programs fit all machines. Thus, only those programs compatible with a particular machine can be considered as available for use with it. A list of commercially available programs and devices may be obtained from the Department of Audio Visual Instruction, National Education Association, 1201 Sixteenth St., N.W., Washington 6, D.C.

3. In evaluating the specific content which a self-instructional program proports to teach, the program can be examined to determine what the student is required to do and whether this reflects the kind of competence which the educator wishes to achieve. Like other educational materials, programs labeled with the name of a particular subject matter vary sidely with respect to content and instructional objectives.

4. Just any set of question-and-answer material does not constitute a self-instructional program. One type of self-instructional material proceeds by small steps requiring frequent student responses. These steps can be examined to see if they embody a careful logical progression of the subject matter. Items in such a program are designed so that the student will respond to the critical aspects of each item or will perform the important operation which that item was meant to teach. Furthermore, such programs generally provide a wide range of examples illustrating each principle or concept.

5. Self-instructional materials are designed to adapt to individual differences by allowing each student to proceed at his own rate. Some types of self-instructional programs further adapt by "branching" to alternate material. For this purpose, questions are designed to diagnose the student's needs and to provide alternate material suited to these needs. The material is designed so that the choice of answer to a particular question determines which items will be presented next. Incorrect answers take the students to items containing information designed to correct the error before continuing through the sequence.

6. An important feature of almost all self-instructional materials is the record of the students' responses, which provides a basis for revising the program. The prospective purchaser should ask about the extent to which revision has been based on student response and how the preliminary try-out was conducted.

7. The effectiveness of a self-instructional program can be assessed by finding out what students actually learn and remember from the program. The prospective purchaser should find out whether such data are available and for what kinds of students, and under what conditions the data were obtained.

8. Active experimentation with self-instructional materials and devices in school systems is to be encouraged prior to large scale adoption.

# GLOSSARY OF TERMS

## APERTURE, EFFECTIVE

The effective aperture of a photographic objective for a given setting of the diaphragm is an opening equivalent to a right section of the largest beam of parallel light from a distant axial objective point that is transmitted by the lens. It is usually circular or approximately so and is specified by its diameter.

## ASPECT RATIO

The ratio of width to height generally used regarding the projected image. May also be applied to a film frame camera printer or projector aperture, a screen or a screen mask.

## BRIGHTNESS

A photometric measure of light emission per unit area of a luminous body or of a translucent or reflecting surface - i. e. candle power per unit area.

## COLOR TEMPERATURE

The temperature to which a black body radiator must be raised in order that the light it emits may match a given light source in color, usually expressed in degrees kelvin (degree K).

## CONTINUOUS TONE

Photographs in which the detail and tone values of the subject are reproduced by an infinite gradation of gray densities between white and black.

## CONTRAST

The general term used to describe the tone separation in a print or viewed image which corresponds to a given difference in the light and shade of the recorded image, or subject from which it was made. Thus contrast is measured by a difference in density or ratio of opacities.

## CONTRAST - COLOR

In color photography, the distinction between colors having equal brightness, but unequal hue and saturation.

## DENSITY

The light absorbing quality of a photographic image usually expressed as the logarithm of the opacity. This concept can be applied to transmission density or to reflectance density.

Light sensitive component of diazo type materials which reacts with couplers to form azio dyes.

DIFFUSION

The scattering of light rays so as to cause the light falling on a surface or passing through an aperture to be coming from all directions in contract to the regular radiation of light from a single point source. Diffusion may be introduced by reflection from a mat surface, by transmission through a frosted or opal glass, or by the use of an integrating bar. When diffusion is complete, a sharp image of the light source can no longer be seen and its place is taken by a uniform extended source which emits light equally in all directions.

FLARE

A defective optical system in which scattered light resulting from reflections at optical surfaces, the barrel of the lens, or imperfections in the optical parts reaches the image plane and produces an over-all fog or flare spot which damages the photographic quality of the resulting image.

HALFTONES

Reproduction of a photograph in which the gradation of tone is reproduced by various microsize dots and intermittent white spaces caused by interposing a screen between the lens and the film.

IMAGE, REAL

In optics, an image which actually exists and could be thrown on a screen in the position where it appears to be.

IMAGE, VIRTUAL

In optics, an image whose existence and properties are inferred by tracing various rays of light backward into or beyond some lens or mirror surface. A virtual image does not exist at the location where it appears to be. However, the rays of light which appear to come from the virtual image behave in the same way that they would behave if they came from a real image having the location of the virtual image and in the absence of any interfering optical surfaces.

INTRINSIC PROGRAM

An approach to teaching machine programming where the student's answering choice may lead him to turn to a particular page in a scrambled book or to press a particular button on a machine at which point a particular predetermined presentation appears on the viewing screen in the machine. This type of responsive programming requires no intermediate computation between the student's answer choice and the decision as to what material he will see next.

MARGINAL RAYS

Rays of light passing through an optical element near the edge of its aperture.

MASTER FILM

Any film, but generally a negative, which is carefully inspected or edited and then used only in making prints. A master film is never used for day-by-day reference.

NEUTRAL WEDGE

Non-selective or colorless optical filter with gradation from low to high density.

QUARTO BOOK SIZE

According to the American Library Association's scale, a book which measures from 25 centimeters to 30 centimeters in height.

RELATIVE APERTURE

The relative aperture is the ratio of the equivalent focal length to the diameter of the effective aperture. The symbol for relative aperture written as a fraction is f/ followed by a numerical value.

SPECULAR

Like a mirror reflecting in a regular manner so that clear images may be formed non-diffusing. The meaning is extended to include regular (non-diffusing) transmission as well as reflection - e. g. specular density as contrasted to diffuse density.